STATISTICS OF VELOCITY DIFFERENCES BETWEEN LAGRANGIAN TRACERS IN A DEVELOPED TURBULENT FLOW

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Abstract The statistics of Lagrangian pair dispersion in a homogeneous isotropic flow is investigated by means of direct numerical simulations. The velocity differences between tracers is found to display a strongly anomalous behavior whose scaling properties are rather close to that of Lagrangian structure functions. Conversely, it is found that the mixed moment defined by the ratio between the cube of the longitudinal velocity difference and the distance attains a statistically stationary regime on short timescales.

The large-time diffusive behaviour of tracers advected by a turbulent flow is commonly used in applications, as for instance in air quality control. Effective mixing properties are modelled in terms of an eddy diffusivity, which is used to assess possible health hazards due to a long exposure downstream a pollutant source. However this approach fails when interested in the likeliness of finding a local concentration exceeding a high threshold. Local fluctuations cannot be determined from the average concentration as they relate to its higher-order moments.

Second-order statistics, such as the spatial correlations of a passive scalar, are related to the relative motion of tracers (see, e.g., [5]). The problem is then to understand the time evolution of the separation $\vec{R}(t) = \vec{X}_1(t) - \vec{X}_2(t)$ between two Lagrangian trajectories. In turbulence, the distance $|\vec{R}|$ follows Richardson's superdiffusive law $\langle|\vec{R}(t)|^2\rangle \sim \varepsilon\, t^3$, where ε is the mean rate of kinetic energy dissipation. The long-term behaviour is thus becoming independent of the initial separation $|\vec{R}(0)| = r_0$, whence the designation of *explosive* pair separation.

The goal of the present work is to give some new phenomenological understanding of the mechanisms leading to this explosive law. For this, we make use of direct numerical simulations. The Navier–Stokes equation with a large-scale forcing is integrated in a periodic domain using a massively parallel spectral solver at a resolution of 4096^3 grid points (see [4] for more details). The flow, which has a Taylor Reynolds number $R_\lambda \approx 730$, is seeded with 10^7 Lagrangian tracers. After a time sufficiently long to have converged to a statistical steady state, we start the analysis of the dispersion of tracer pairs. For this, we label at an arbitrary initial time (that we fix here to be $t = 0$) all couples whose distance $|\vec{R}(0)|$ is equal to $r_0 \pm 2\%$ for $r_0 \geq 8\eta$ in the inertial range.

We focus here on the statistics of the longitudinal component $V_\parallel(t) = \vec{R}(t)\cdot\vec{V}(t)/|\vec{R}(t)| = \mathrm{d}|\vec{R}(t)|/\mathrm{d}t$ of the velocity difference $\vec{V}(t)$ between the tracers. Richardson's explosive law suggests that at sufficiently long times, $V_\parallel(t)$ behaves as $t^{1/2}$. This is confirmed numerically. We indeed observe in Fig. 1 (Left) that after a initial ballistic increase $\propto t$, one has $\langle V_\parallel(t)\rangle \propto t^{1/2}$ (red curve) on nearly one decade at sufficiently large times. The time of convergence to this

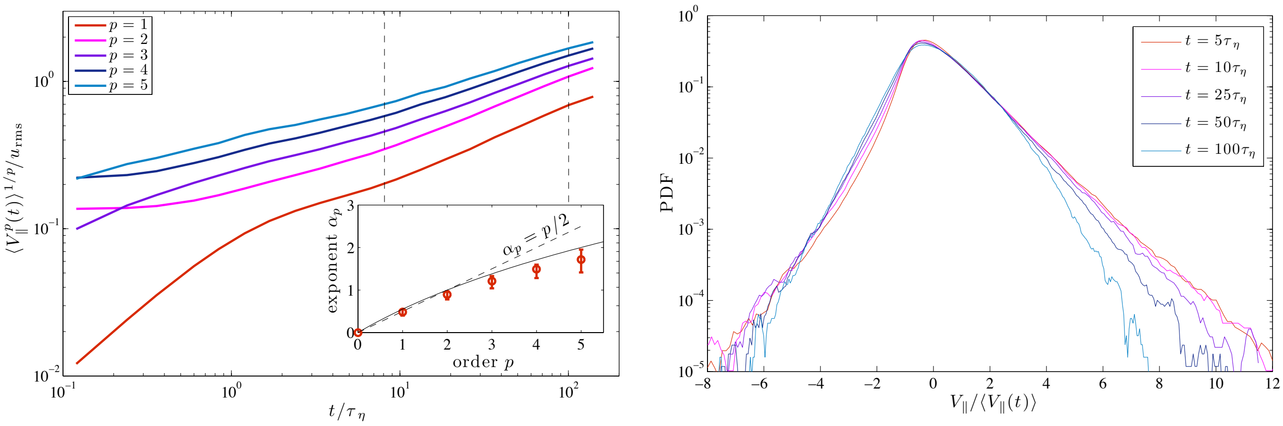

Figure 1. (Left) Time evolution of the moments of the longitudinal velocity difference $V_\parallel(t)$ for various orders as labeled and for $r_0 = 8\eta$. Inset: exponents α_p (red symbols) as a function of the order estimated from the average local slope in the region delimited by the two dashed lines in the main figure; the dashed line corresponds to a self-similar behaviour of $V_\parallel$ and the solid line to the multifractal prediction for the exponent of the Lagrangian structure function [1]. (Right) Probability density function (PDF) of $V_\parallel$ for $r_0 = 8\eta$ and at various times as labeled.

behaviour depends on r_0 as it is of the order of $t_0 = S_2(r_0)/(2\varepsilon)$, where S_2 is the second-order structure function with absolute values (see [2]). As seen in Fig. 1 (Left), higher-order moments of $V_\parallel$ also increase algebraically in time, namely $\langle V_\parallel^p(t)\rangle \propto t^{\alpha_p}$. As seen in the inset, these exponents could be measured numerically using the average local slope of the moments as a function of time over a bit more than a decade (from $8\tau_\eta$ to $100\tau_\eta$). On finds that $\alpha_1 \approx 1/2$ but $\alpha_p < p/2$

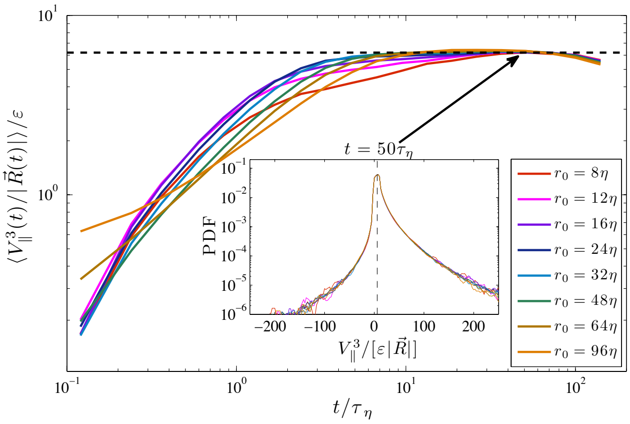

Figure 2. Time evolution of the average local rate of kinetic energy transfer $\langle V_\parallel^3(t)/|\vec{R}(t)|\rangle$ for various initial separations between the particles. Inset: Probability density function of $V_\parallel^3(t)/|\vec{R}(t)|$ at time $t = 50\tau_\eta$ and for the same initial separations.

when $p > 1$, which is a signature of an intermittent behaviour. The dependence of α_p on the order p is very close to that of the exponents of Lagrangian structure function found in [1]. We have measured these exponents for other values of the initial separation r_0 between the tracers and obtained the same results. This intermittent behaviour in time of the moments can also be observed in Fig. 1 (Right) for the probability density functions of $V_\parallel$ measured at several times in the scaling regime. A self-similar behaviour would be evidenced by a collapse of the various curves. This is clearly not the case as one observes substantial dependences on time in the right-hand tail and at moderate negative values.

The velocity difference between tracers thus displays very intermittent features and, as a consequence, does not converge to a behaviour with temporal self-similarity, or does it only very slowly. The situation is very different when interested in mixed statistics between distances and longitudinal velocity differences. As seen in Fig. 2, the moment $\langle V_\parallel(t)^3/|\vec{R}(t)|\rangle$, which is initially negative and equal to $-(4/5)\epsilon$, tends very quickly to a positive constant. This quantity, which can be interpreted as a local rate of energy transfer along the tracer paths, is thus conserved by the Lagrangian flow at sufficiently long times. Note that the slight decrease observed at very large times comes from the contamination of the statistics by pairs that have reached a distance of the order of the integral scale.

Actually, as stressed in [3] it is not only the average of the "local transfer rate" that converges to a constant but its full distribution seems to attain a stationary regime on rather short timescales. The inset of Fig. 2 shows the PDFs of $V_\parallel^3/|\vec{R}|$ for different initial separations and at a fixed time ($50\tau_\eta$) sufficiently large to be ensured that all distributions have attained their asymptotic regime and sufficiently small to assert that there is no contamination by pairs having reached the flow large scales. One observes a robust collapse, much more pronounced than for both the distribution of separations and that of velocity differences. The asymptotic probability distribution is peaked around zero (rather than its mean value), asymmetric, and displays fat tails that, according to our data, are $\propto \exp(-C\,|V_\parallel|/|\vec{R}|^{1/3})$ on both sides.

To our knowledge, the convergence of these mixed statistics to a stationary regime has never been reported in other numerical or experimental investigations. From a phenomenological viewpoint, one expects in Richardson's regime $V_\parallel \propto t^{1/2} \propto |\vec{R}|^{1/3}$, so that the local transfer rate $V_\parallel^3/|\vec{R}|$ should become constant at sufficiently large times. However we observe that the convergence of this quantity to its asymptotic value occurs much faster and in a much more definite manner than the convergence of the statistics of $\vec{R}$ and $\vec{V}$ to their respective asymptotic forms. Also, the local transfer rate seems much less intermittent than the velocity. All these considerations suggest that the statistical stationarity of $V_\parallel^3/|\vec{R}|$ is more likely to be a cause rather than a consequence of Richardson's explosive separation. Unfortunately, we presently lack a clear understanding of the underlying physical explanation for this behaviour.

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