WHAT RDT TELLS US ABOUT T/NT INTERFACES

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Abstract The boundary layer separating a shear-free (no mean shear) isotropic turbulence region from an irrotational (or non-turbulent) flow region is investigated using rapid distortion theory (RDT), assuming that the turbulent/non-turbulent (T/NT) interface is suddenly inserted and remains approximately flat. When only inviscid processes are taken into account, the values taken at the T/NT interface by turbulence statistics, like the TKE and the dissipation rate ε, do not depend on the form of the turbulence energy spectrum. The dissipation rate decays like z^{-6} in the irrotational flow region. When viscous effects are taken into account, the agreement of various turbulence statistics with DNS data is found to be very good. ε displays a large maximum at the T/NT interface, which can be quantitatively related to the thickness of the viscous boundary layer (VBL). For an equilibrium VBL, RDT suggests that the T/NT interface thickness scales with the Kolmogorov microscale.

INTRODUCTION

The scaling for the thickness of T/NT interfaces, separating turbulent from laminar flow, and the dynamics of fluctuating quantities near such interfaces, have been the object of recent interest [3, 2, 6]. These aspects are relevant for understanding how entrainment and mixing occurs across these interfaces, and how turbulence consequently expands. In this study the simplest possible T/NT interface, separating homogeneous and isotropic shear-free turbulence from initially quiescent fluid, is addressed using RDT [5]. The interface is assumed to be suddenly inserted and to remain flat (at $z = 0$) as the flow adjusts between the two regions of the fluid.

THEORETICAL MODEL

In situations without mean shear, as is the case here, RDT is formally valid for times since boundary insertion shorter than one eddy turn-over time. The equations of motion may then be linearized [1]. The total turbulent velocity is split into three components: a homogeneous and isotropic velocity $\mathbf{u}^{(H)}$ (existing only for $z > 0$), an irrotational velocity component $\mathbf{u}^{(S)} = \nabla\phi^{(S)}$ and a viscous velocity component $\mathbf{u}^{(V)}$, both existing either side of $z = 0$. It turns out that $\phi^{(S)}$ satisfies Laplace's equation, and the viscous velocity components tangent to the interface satisfy diffusion equations. Additionally, both $\phi^{(S)}$ and $\mathbf{u}^{(V)}$ must decay to zero as $z \to \pm\infty$ and the total velocity and its derivative normal to the interface, as well as the pressure, must be continuous at $z = 0$.

All flow variables are expressed as Fourier integrals along the directions tangential to the interface, and statistics of the fluctuating velocity are calculated, assuming that the energy spectrum of the isotropic and homogeneous turbulence existing at $z \to +\infty$ takes the form

$$E(k) = \frac{q^2\lambda_\infty}{(2\pi)^{1/2}}(k\lambda_\infty)^4 e^{-(k\lambda_\infty)^2/2}, \tag{1}$$

where k is the wavenumber, q is the root-mean-square velocity and λ_∞ is the Taylor microscale of this turbulence.

RESULTS

Figure 1 shows profiles of the TKE, $K = (1/2)(\overline{u^2} + \overline{v^2} + \overline{w^2})$ (Fig. 1a), and of the dissipation rate ε (Fig. 1b), normalized by their values deep inside the isotropic turbulence, as a function of distance from the interface normalized by λ_∞, ignoring the viscous boundary layers. This would correspond to a time immediately after boundary insertion. Both K/K_∞ and $\varepsilon/\varepsilon_\infty$ decay to zero in the non-turbulent zone ($z < 0$) and take values at $z = 0^+$ and $z = 0^-$ that do not depend on the form of the assumed energy spectrum (1).

Figure 2 shows the normalized dissipation rate when viscous effects are taken into account, for a time at which $\delta/\lambda_\infty = 0.1$ (where δ is the VBL thickness). It can be seen in Fig. 2a that ε displays a very large maximum at $z = 0$, which is due to the initial discontinuity of the tangential velocity components. It can be shown that this maximum is given by

$$\frac{\varepsilon(z=0)}{\varepsilon_\infty} \approx \frac{2}{15\pi}\left(\frac{\delta}{\lambda_\infty}\right)^{-2}. \tag{2}$$

So, the scaling for the VBL thickness (which can be identified with the T/NT interface thickness) is related to the ε maximum, as long as the VBL remains sufficiently thin. Figure 2b shows the asymptotic behaviour of ε in the irrotational flow region. It can be seen that $\varepsilon/\varepsilon_\infty \sim (z/\lambda_\infty)^{-6}$ as $z \to -\infty$, a result that parallels that of [4] for K/K_∞.

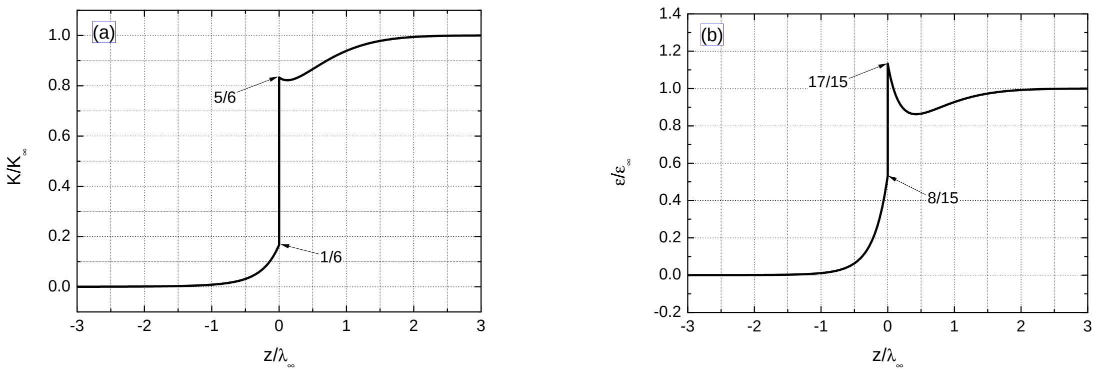

Figure 1. Inviscid results. Profiles of the normalized (a) TKE and (b) dissipation rate as a function of normalized distance from the T/NT interface.

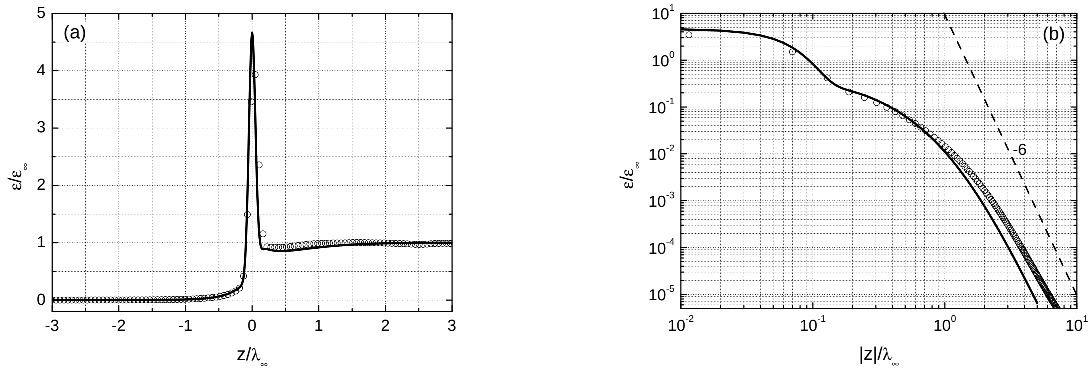

Figure 2. Viscous results. Normalized disspation rate for $\delta/\lambda_\infty = 0.1$ (a) as a function of distance from the T/NT interface, and (b) its asymptotic behaviour in the non-turbulent region $z < 0$. Lines: RDT, symbols: DNS [5].

Real turbulent flows are generally found to be described approximately by RDT taken at a time close to its limit of validity due to the growing importance of the neglected nonlinear effects. If this time is assumed to be $t = L_\infty/q$, where L_∞ is the integral length scale of the isotropic turbulence at $z \to +\infty$, then the definition of the VBL $\delta = (\nu t)^{1/2}$ (where ν is the molecular viscosity) yields $\delta = (\nu L_\infty/q)^{1/2}$, which implies $\delta \sim \lambda_\infty$. If, on other hand (and as seems to make more sense physically) [5], the time limit for RDT is

$$t = \lambda_\infty/q \quad \text{then} \quad \delta = (\nu\lambda_\infty/q)^{1/2} \sim \eta, \tag{3}$$

where η is the Kolmogorov microscale. This last result seems to be more consistent with previous DNS results.

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