ON PSEUDO SELF-SIMILAR REGIMES IN ISOTROPIC TURBULENCE DECAY

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Abstract The issue of the evolution of self-similar regimes in homogeneous isotropic turbulence (HIT) is addressed by the use of an eddy-damped quasi-normal Markovian (EDQNM) model. The numerical results show that a complete self-similar regime is not universal and is observed for specific initial conditions only. Moreover, the analysis of a three range energy spectrum indicates that the shape of the energy spectrum $E(k,t)$ close to its peak drive the emergence of different regimes. The very large scales for $E(k,t) \to 0$ have a negligible impact over the decay laws followed by the HIT statistical quantities.

INTRODUCTION

In the present work, the issue of the observation of self-similar regimes in HIT decay is addressed. This subject has been extensively investigated in open literature and a number of definitions of self-similarity have been proposed in the years. The oldest proposals in literature state that in a self-similar regime the statistical quantities related to the turbulent flow, such as the turbulent kinetic energy u^2, the dissipation rate ε and the characteristic length scale l, decay following a power law. In another well known proposal, the energy spectrum $E(k,t)$ can be univocally described by the expression:

$$E(k,t) \propto u^2(t)\, l(t)\, F(k, l(t)) \tag{1}$$

where $F(k, l(t))$ is a time independent shape function. This latter definition is compatible with the first one. In fact, Comte–Bellot & Corrsin[1] showed that a power law decay of the physical quantities is recovered if the energy spectrum is described by Equation 1. In particular, the power law coefficients of the decay laws are governed by the slope of the energy spectrum at the large scales. Other controversial definitions have been reported in open literature. One deals with the existence of a universal regime for which the integral scale L collapses with the Taylor microscale λ, and for which $u^2 \propto t^{-1}$. This regime is referred to as completely self-similar. Accordingly to some studies, this state can be observed at asymptotically infinite Reynolds numbers[2] or after a sufficiently long decay time[3, 4]. Experiments and numerical simulations have not been able to completely assess these theoretical models, and a univocal definition of self-similarity is elusive.

By the use of an EDQNM model, the HIT free decay is systematically investigated in the range of parameters $\sigma \in [1,\, 4]$, $10^{-3} \leq Re_\lambda \leq 10^5$. This model, which is based on a spectral discretisation of the Lin equation:

$$\frac{\partial E(k,t)}{\partial t} + 2\nu k^2 E(k,t) = T(k,t) \tag{2}$$

where k is a spectral element, t the characteristic time, ν the kinematic viscosity and $T(k,t)$ the non linear energy transfer, is an excellent candidate to study HIT statistical quantities[5].

COMPARISON OF THE EDQNM RESULTS WITH THE THEORETICAL FRAMEWORKS REPORTED IN LITERATURE

The numerical results indicate that the complete self-similar regime previously introduced is a particular case of the framework proposed by Comte–Bellot & Corrsin[1], for $\sigma = 1$. Indeed, the decay law $u^2 \propto t^{-1}$ is never observed for $\sigma \neq 1$, in the Re_λ range investigated and for very long decay times. As a consequence, a complete scaling of the energy spectrum on a singular length scale, such as the one proposed by George[2], is recovered for the case $\sigma = 1$ only. In the other cases, a partial scaling on the spectrum is observed, which is relative to the characteristic length considered.

Moreover, the issue of the observation of self-similarity from the analysis of the HIT statistical quantities is addressed. To this aim, following the work by Comte-Bellot & Corrsin[1], a generalised theoretical model is formulated to describe homogeneous isotropic turbulence (HIT) decay with composite three-range spectrum:

$$E(k) = \begin{cases} A\, k^{\sigma_1} & kl_1 \ll 1 \\ B\, k^{\sigma_2} & kl_1 \gg 1,\; kl_2 \ll 1 \\ C_k \varepsilon^{2/3} k^{-5/3} & kl_2 \gg 1 \end{cases} \tag{3}$$

The theoretical analysis takes into account the presence of different ranges to describe the large scales in the energy spectrum formulation. Thus, two characteristic lengths l_1 and l_2 are initially imposed. Thanks to dimensional analysis, it is possible to predict the time evolution of different decaying regimes. The complete formulae are reported in Table 1 for

	n_{u^2}	n_l	n_ε
$t < t_c$	$2(\sigma_2 - p_2 + 1)/(\sigma_2 - p_2 + 3)$	$2/(\sigma_2 - p_2 + 3)$	$(3(\sigma_2 - p_2) + 5)/(\sigma_2 - p_2 + 3)$
$t > t_c$	$2(\sigma_1 - p_1 + 1)/(\sigma_1 - p_1 + 3)$	$2/(\sigma_1 - p_1 + 3)$	$(3(\sigma_1 - p_1) + 5)/(\sigma_1 - p_1 + 3)$

Table 1. Analytical formulae recovered by the theoretical approach derived from the analysis of the three-range energy spectrum, in the case of a finite value of the critical time t_c.

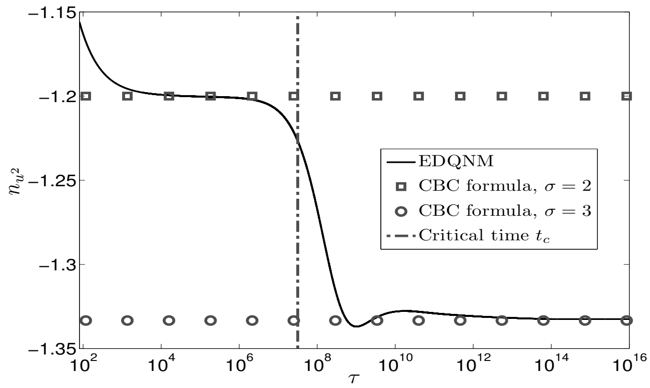

Figure 1. Time evolution of the power law exponent relative to turbulent kinetic energy. A composite three-range energy spectrum is initially imposed at $Re_\lambda = 10^5$ for $\sigma_1 = 3$ and $\sigma_2 = 2$.

the physical quantities of interest. The coefficient $p_i = max(0, \sigma_i - 3.2)$ is a correction coefficient which represents the effects of the non local energy transfer[6]. The theoretical model indicates that, if the permanence of large eddies (PLE) hypothesis is verified, the turbulent flow initially evolves through a pseudo self-similar regime which is governed by the parameter σ_2. This parameter represents the slope of the energy spectrum at the most energetic scales. After a critical time t_c, the self-similar regime will be governed by the parameter σ_1, which represents the slope of the energy spectrum at the very large scales. Thus, the decay regime will initially be not self-similar, due to the presence of two different length scales. But, it will behave *as if* it was self-similar, if the observation of the decaying regime characteristics is limited to the power law exponents of the relevant physical quantities of the flow. This situation is exemplified in Figure 1, where EDQNM results are reported to assess the theoretical framework. The turbulent kinetic energy power law exponent n_{u^2}, $u^2 \propto t^{n_{u^2}}$ is shown. The evolution of the decay regime is more complex if at least one of the two ranges initially enforced does not verify the PLE hypothesis, which corresponds to the relation $\sigma_i > 3.2$. In fact, depending on the initial conditions, the decay regime can evolve to a pseudo self-similar regime (finite t_c) or to a lasting non self-similar regime (infinite t_c).

The main conclusion drawn from the present work, which results are extensively discussed in the recent work by Meldi & Sagaut[6], is that the characteristics of the decaying regime are governed by the shape of the energy spectrum close to its peak only. Thus, it is not possible to determine the self-similarity by sole observation of the characteristics of the decay regime. This result is not in agreement with most of the theoretical analysis reported in literature, which predict a strong dependence of the decay regime evolution from the shape of the energy spectrum for $k \to 0$.

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