TRANSITION TO TURBULENCE IN THE ROTATING-DISK BOUNDARY LAYER

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Abstract The development of the flow over a rotating disk is investigated by direct numerical simulations using both the linearised and fully nonlinear Navier–Stokes equations. The nonlinear simulations allow investigation of the transition to turbulence of the realistic spatially-developing boundary layer, and these simulations can be directly validated by physical experiments of the same case. The current research aims to elucidate further the global stability properties of the flow. So far, there are no conclusive simulations available in the literature for the fully nonlinear case for this flow, and since the nonlinearity is particularly relevant for transition to turbulence an increased understanding of this process is expected.

INTRODUCTION

We study the incompressible boundary layer over a rotating disk without any imposed flow. The laminar profiles arising over the disk are seen in Fig. 1 and constitute the similarity solution of the cylindrical Navier–Stokes equations for a disk of infinite radius. The boundary layer thus consists of a three-dimensional axisymmetric flow, with a constant thickness and a Reynolds number increasing linearly with radius. The radial velocity component (U) is inflectional making the flow susceptible to an inviscid instability. The stability properties have long been examined both through experiments and theory, *e.g.* Refs. [10, 6], yet there is no clear picture of the precise mechanism leading to turbulent flow.

Seventy-four years after von Kármán in 1921 derived the similarity solution [9], a local spatio-temporal stability analysis of the governing equations revealed an absolute instability [6]. For a spatially-varying flow, an absolute instability is a local concept in that it is defined theoretically by a stability analysis of the profiles at the locally prescribed Reynolds number. The theoretical critical Reynolds number found for the onset of the absolute instability agrees well with the experimentally observed location for the onset of nonlinearity and the transition process [7]. In 2003, the first DNS of the linearised Navier–Stokes equations was conducted for this flow case and showed that there is no evidence of the local absolute instability giving rise to a global oscillator [1]. This DNS work is of particular interest because it relates to the single rotating-disk case and is one of very few examples known to the authors to consider global stability.

Davies and Carpenter [1] neglected inwardly-traveling disturbances from the outer radial boundary; these disturbances are, however, fundamental to the absolute-instability mechanism [6]. The results presented in Ref. [1] have led to further work, *e.g.* Refs. [8, 4, 5], to determine whether their finding of linear global stability is a product of the linear approximation and also to determine whether the effect of the disk's finite radius plays a significant role in physical experiments. Inclusion of nonlinearity [8] has shown that this can lead to global instability, and it has been hypothesised [4] that the finite radius of the disk can lead to a linear global instability. To continue the investigation of the role of the absolute instability, it is highly relevant to have a reliable numerical set-up for the case to allow for accurate simulations of the spatially-evolving flow. DNS results have been validated against experimental results, showing that our DNS approach is suitable for analysing the transition process further.

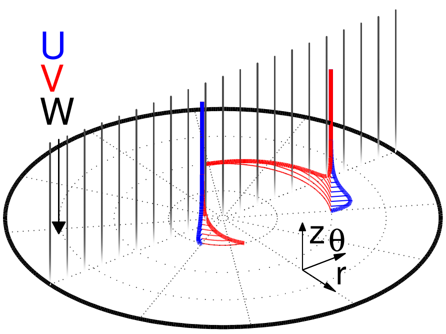

Figure 1: The laminar velocity profiles of the similarity solution for the flow over a rotating disk. U is the radial velocity component, V is the azimuthal velocity component and the vertical greyscale lines indicate the profile for W (white is zero velocity).

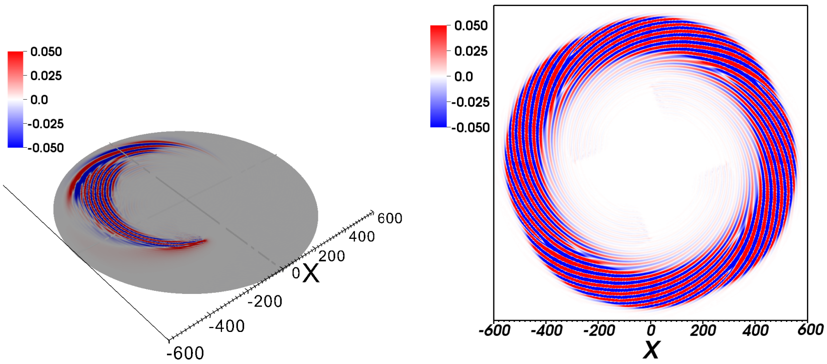

Figure 2: Snapshots of the azimuthal velocity perturbations for surface roughnesses where time, T, corresponds to radians of the disk rotation. The coloured plane shown is $z = 1.3$ where z is the non-dimensional height from the wall ($z = z^*(\Omega^*/\nu^*)^{1/2}$, where Ω^* is the dimensional rotation rate and ν^* is the dimensional kinematic viscosity). X refers to the X-coordinate on the cartesian grid, corresponding to the Reynolds number measured from the plane centre. *Left*: simulation made in the inertial frame of reference containing a single large roughness element, $T = 4.02$. *Right*: simulation made in the rotating frame of reference containing four small roughness elements, $T = 7.54$.

SIMULATIONS OF THE BOUNDARY LAYER OVER A ROTATING DISK

Our simulations are performed with the massively parallel spectral-element code nek5000 [3]. nek5000 is a Legendre polynomial based Spectral Element Method code which solves the incompressible Navier–Stokes equations. The temporal discretization is based upon operator splitting, where the nonlinear convective terms are treated explicitly via a projection scheme, and the viscous and divergence operators are treated implicitly [2]. The spectral elements within the code are flexible and can be used to build complex geometries.

When validating our set-up, the laminar flow was simulated and the profiles corresponded to those of the von Kármán flow; see Fig. 1. Also, the perturbations to the mean flow produced by adding stationary (relative to the disk surface) roughness elements on to the disk have been calculated. The perturbation fields are seen for two different simulations in Fig. 2 for roughness elements introduced at radius 290 $(\nu^*/\Omega^*)^{1/2}$. Left of Fig. 2 shows a simulation conducted for one roughness element when the flow is still developing. After approximately 2/3 rotation of the disk (4.02 radians) a trail of convective disturbances propagating towards the edge of the disk is created from the roughness. Right of Fig. 2 shows a simulation conducted in the rotating reference frame containing four smaller roughness elements. They have created a pattern of stationary vortices around the disk with the number corresponding to observations in experiments [10].

In the final contribution, we will present fully resolved simulations with stationary roughness elements and time-dependent disturbances, which will allow us to analyse the absolutely-unstable region in more detail. Here, both linear and nonlinear simulations are of particular importance to distinguish when, where and how the nonlinear effects enter the flow. We will compare our results with those of Refs. [1] and [8]. We will also look at the influence of the edge of the disk to compare with recent results in Refs. [4] and [5]. In the latter case there are two ways of doing so: by considering the numerical boundary incorporated within the model; or by introducing a physical edge of the disk within the simulations.

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