PASSIVE SCALAR MIXING: TURBULENCE VERSUS CHAOTIC ADVECTION.

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Abstract Turbulent transport and mixing of a passive scalar blob in a confined vessel are studied. The mixing process is due to the fluid dynamics generated by a single rod with a figure-eight shaped motion. A volume penalization method in a classical Fourier pseudo-spectral code is used to define the confined vessel and to impose the dynamical motion of the rod. The two-dimensional incompressible Navier-Stokes and advection-diffusion equations are solved and its dynamics are compared to advection-diffusion by Stokes flow. The decay of scalar variance in Stokes regimes, for different Schmidt numbers, is compared with the one obtained in Ref. [1] for chaotic mixing. Afterwards, the influence of Reynolds and Schmidt numbers onto turbulent mixing is investigated. The existence of power-laws for time evolution of the scalar variance is particularly underlined. In turbulent regimes, the mixing is found to become more efficient for increasing Reynolds and Schmidt numbers. The product of these two numbers, the large scale Péclet number, does not seem the only parameter which controls the mixing rate.

INTRODUCTION

The present investigation focuses on the role of the nonlinear term of the Navier-Stokes equations on the mixing efficiency. In particular will we focus on the difference in behavior between chaotic advection and nonlinear mixing, i.e., the advection-diffusion dynamics in a flow field which contains, or not, the nonlinear advection term in the Navier-Stokes equations. The flow geometry we consider is one which has been extensively considered in studies on chaotic advection (see reference [1] for example): two-dimensional incompressible flow in a circular domain in which the fluid motion is generated by a rod, describing an eight-shaped motion with a constant velocity U_Ω. Two regimes are considered. The first one is the Stokes regime in which $\omega \cdot \nabla \vec{u} = 0$ and the second one the nonlinear regime. Different Reynolds number ($Re = DU_\Omega/\nu$, where U_Ω is the velocity of the rod and D is the diameter of the rod) and Schmidt number ($Sc = \nu/\kappa$) are studied. This definition based on the rod velocity allows to define a Reynolds number in the Stokes regime, even though, according to its physical definition, the Reynolds number is strictly zero in Stokes flow. The initial condition for the scalar is a Gaussian blob placed along the axis center, and the flow is initially at rest. The two-dimensional incompressible Navier-Stokes and advection-diffusion equations are solved using direct numerical simulation and a volume penalization method is applied to impose no-slip boundary conditions for velocity and no-flux for the scalar. More details on the numerical methods can be found in Ref. [2, 3].

SCALAR MIXING RESULTS

Figure 1 shows snapshots of vorticity and scalar fields. For Stokes regimes, the scalar remains confined to a circular region, leaving a part of the domain in the vicinity of the walls unmixed. This effect has been explained by Gouillart et al. [1] to negatively influence the mixing efficiency. In the turbulent regime the scalar is spread into the entire fluid domain. Indeed, in the nonlinear regime, vortices are created by the detachment of the boundary layer, generated by the motion of the rod. These vortices are then injected into the bulk flow. This vortical motion leads to an enhanced mixing. An important issue, which will be addressed in the full paper, is how the energy injected in the system relates to the mixing efficiency. In the present abstract we will simply quantify the decay of scalar variance for different flows, without taking into account the energy input needed to obtain this mixing.

Time evolutions of kinetic energy of the flow and of scalar variance are plotted in Figure 2. The kinetic energy increases with increasing Reynolds number, which is due to the fact that for higher Reynolds number, more vortical motion is created, which persists for a longer time. For long times, the scalar variances exhibit a power law behavior. For intermediate times $30 > t > 10$, the scalar variances decay as a power law close to t^{-3}. The slowest decay of scalar variance happens for the Stokes regime. For turbulent regimes, the decay is stronger for increasing $Re \times Sc$ and is much more important for the longest times $t > 30$.

We have performed simulations of scalar mixing in confined domains for different Re and Sc numbers and in particular for Stokes flow and Navier-Stokes flow. Not so surprisingly, "nonlinear" mixing is more efficient than "Stokes" mixing. Its efficiency depends not only on the product $Re \times Sc$. Important issues that will be addressed in the full paper are the mixing efficiency and how it depends on the injected energy in both regimes. Further will we attempt to define dimensionless quantities which characterize the mixing in both regimes.

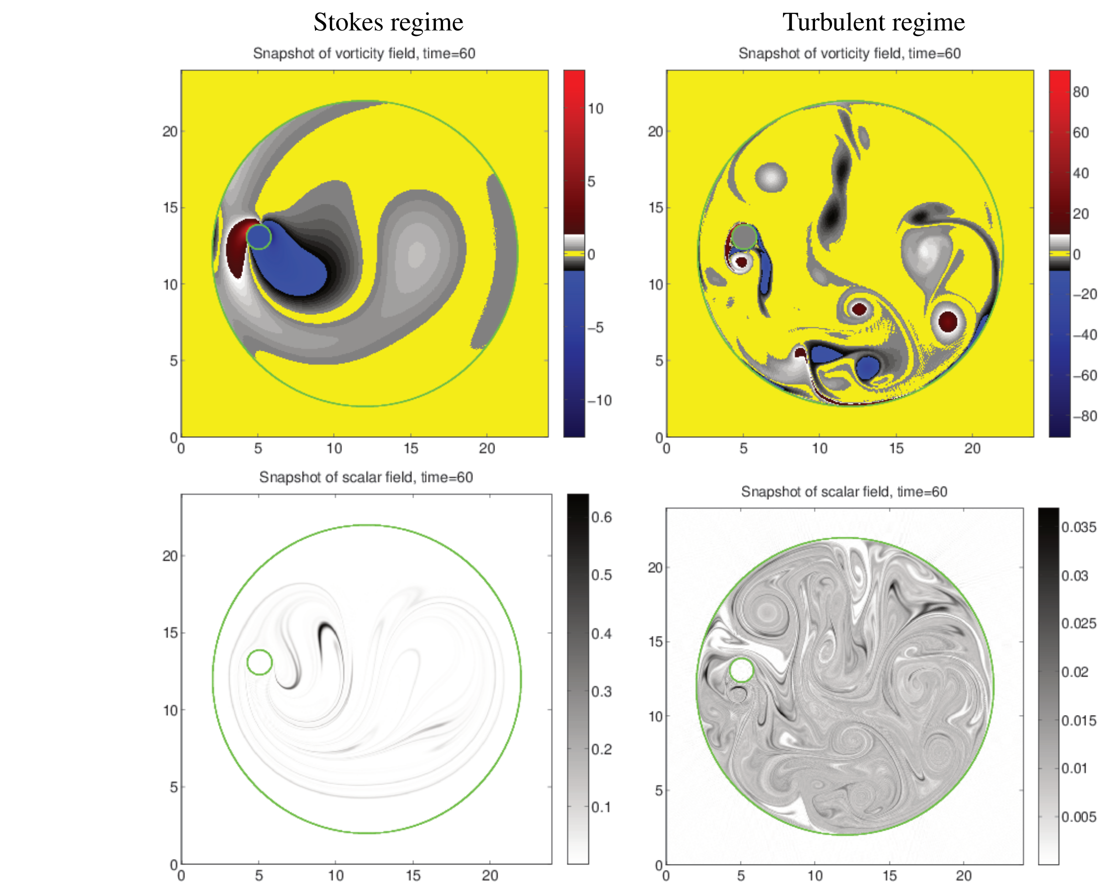

Figure 1. Snapshots of vorticity fields (top) and of scalar fields (bottom) in the Stokes regime $Re = 1$ and $Sc = 10000$ (left) and in a turbulent regime $Re = 100$ and $Sc = 100$ (right).

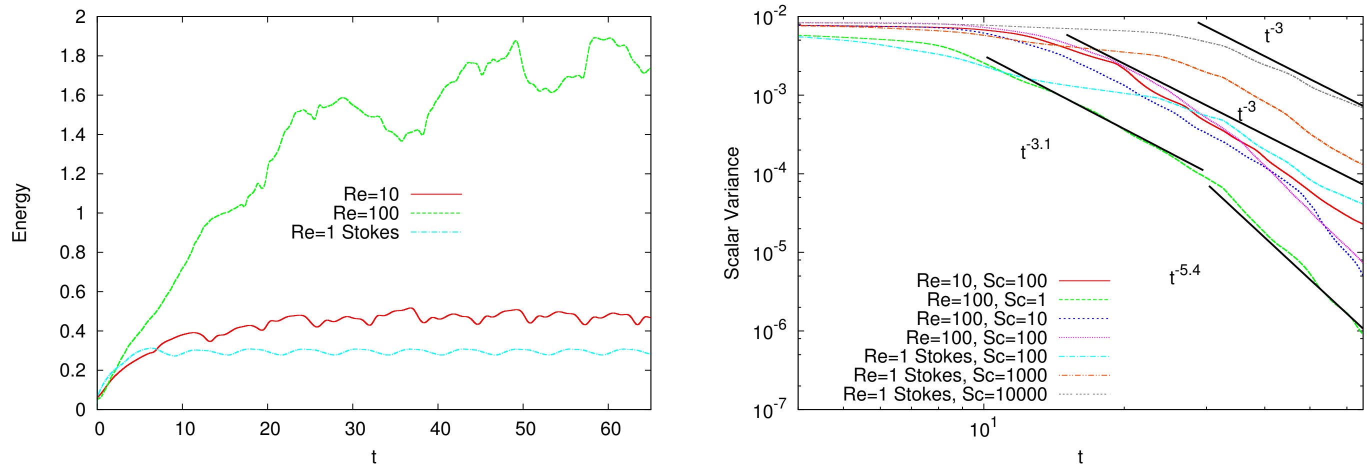

Figure 2. Time evolution of kinetic energy (left) and of scalar variance in log-log representation (right).

References

[1] E. Gouillart, O. Dauchot, B. Dubrulle, S. Roux, and J.-L. Thiffeault. Slow decay of concentration variance due to no-slip walls in chaotic mixing. *Phys. Rev. E* **78**: 026211, 2008.

[2] D. Kolomenskiy and K. Schneider. A Fourier spectral method for the Navier-Stokes equations with volume penalization for moving solid obstacles. *J. Comput. Phys.* **228**: 5687–5709, 2009.

[3] B. Kadoch, D. Kolomenskiy, K. Schneider and P. Angot. A volume penalization method for Navier–Stokes flows and scalar advection-diffusion with moving obstacles. *J. Comput. Phys.* **231**: 4365-4383, 2012.