MODELING OF TURBULENCE ATTENUATION IN PARTICLE- OR DROPLET-LADEN FLOWS

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Abstract Particle- or droplet-laden turbulent flows play a role in many industrial and environmental applications like spray combustion or cloud formation. For numerical simulations of such flows, RANS-based simulation models are applicable. In this work [7], we outline a new model that accurately reproduces particle-induced attenuation of fluid-phase turbulence. The new model is based on a Lagrangian description for the particle-phase, which provides a closure model for particle-phase terms in the fluid-phase RANS and Reynolds stress equations. Moreover, it accounts for preferential concentration effects that become important for Stokes numbers around 1. The model performance is validated for a range of different particle Stokes numbers and particle mass loadings.

INTRODUCTION

For particle volume loadings $\Phi_v = 10^{-5} \ldots 10^{-2}$, the particle-phase has an effect on the fluid-phase turbulence but particles do not interact with each other. An important turbulence modulation effect is turbulence attenuation caused by particles with densities ρ_p that are higher than the fluid density ρ. Eaton [4] has reviewed several experimental and numerical studies where this effect was investigated. For example, Boivin and coworkers [2] have conducted a detailed numerical study, where turbulence attenuation was analyzed for particle mass loadings $\Phi_m = 0 \ldots 1$ and Stokes numbers $\mathrm{St} \equiv \tau_p/\tau_\eta = 1 \ldots 10$. Turbulence attenuation processes are significantly influenced by zones of preferential particle concentrations that are forming for heavy particles with $\mathrm{St} \approx 1$ accumulating in regions of low vorticity [3].

In this work, we propose an elegant Lagrangian particle-phase model that accounts for preferential concentration effects and accurately reproduces fluid-phase turbulence attenuation as observed by Boivin et al. for the entire mass loading and Stokes number ranges considered.

MODEL FORMULATION

The equations of motion of a particle with index n, position $\mathbf{x}^n(t)$, velocity $\mathbf{v}^n(t)$, and mass m_p in a flow field $\mathbf{u}(\mathbf{x},t)$ are given by

$$\frac{\mathrm{d}\mathbf{x}^n}{\mathrm{d}t} = \mathbf{v}^n \quad \text{and} \quad m_p \frac{\mathrm{d}\mathbf{v}^n}{\mathrm{d}t} = \underbrace{-\, m_p \frac{c}{\tau_p}[\mathbf{v}^n - \mathbf{u}(\mathbf{x}^n, t)]}_{\equiv \mathbf{F}^n} + m_p \frac{\rho}{\rho_p} \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} + m_p \left(1 - \frac{\rho}{\rho_p}\right) \mathbf{g} \tag{1}$$

[1, p.123]. In a RANS context, the instantaneous fluid-phase flow field $\mathbf{u}(\mathbf{x},t)$ at position $\mathbf{x}$ and time t is unknown, but the mean $\langle \mathbf{u} \rangle$ and the Reynolds stress tensor U with elements $\langle u_i' u_j' \rangle$ are calculated with the RANS and Reynolds stress transport equations.

We model the instantaneous fluid-phase velocity at the location of particle n, i.e. $\mathbf{u}(\mathbf{x}_p, t)$ referred to as seen fluid velocity, by statistically independent random diffusion processes

$$\mathrm{d}\xi_i^n = -\frac{\mathrm{d}t}{T_L^*}\xi_i^n + \sqrt{\frac{2}{T_L^*}}\,\mathrm{d}W_i \tag{2}$$

for each spatial direction i with correlation timescale T_L^* and Wiener process increment $\mathrm{d}W_i$. To ensure consistency of the modeled seen velocity statistics with $\langle \mathbf{u} \rangle$ and U from the RANS solution, we introduce the transformation

$$\mathbf{u} = \langle \mathbf{u} \rangle + \mathsf{V}\,\mathbf{w}'^n \quad \text{with components} \quad w_i'^n = \sqrt{\langle w_i'^2 \rangle}\,\xi_{(i)}^n \quad \text{and} \quad \mathsf{W} = \begin{pmatrix} \langle w_1'^2 \rangle & 0 & 0 \\ 0 & \langle w_2'^2 \rangle & 0 \\ 0 & 0 & \langle w_3'^2 \rangle \end{pmatrix}, \tag{3}$$

which is based on the diagonalization $\mathsf{V}^T\mathsf{U}\mathsf{V} = \mathsf{W}$ of the Reynolds stress tensor U. As is sketched in Figure 1(c) for a two-dimensional case, the mean velocity $\langle \mathbf{u} \rangle$ and the Reynolds stress tensor U vary spatially. Accordingly, as a particle n travels through the computational domain with $\boldsymbol{\xi}(t)^n$ evolving as given by equation (2), it visits different RANS solution grid cells with changing transformations V and diagonal Reynolds stress tensors W.

The modeling of the correlation time scale T_L^* in equation (2) depends on the Stokes number. For $\mathrm{St} \to 0$, particles behave like fluid particles and T_L^* becomes equal to the Lagrangian velocity correlation time scale of fluid particles. For ballistic particles with $\mathrm{St} \to \infty$, the Eulerian correlation length scales of the flow field become relevant. At intermediate Stokes numbers, He, Jung et al. [5, 6] have shown that preferential concentration effects have a significant influence on T_L^*. In this work, this effect is taken into account through an extended empirical expression for $T_L^*(\mathrm{St})$ based on [5, 6].

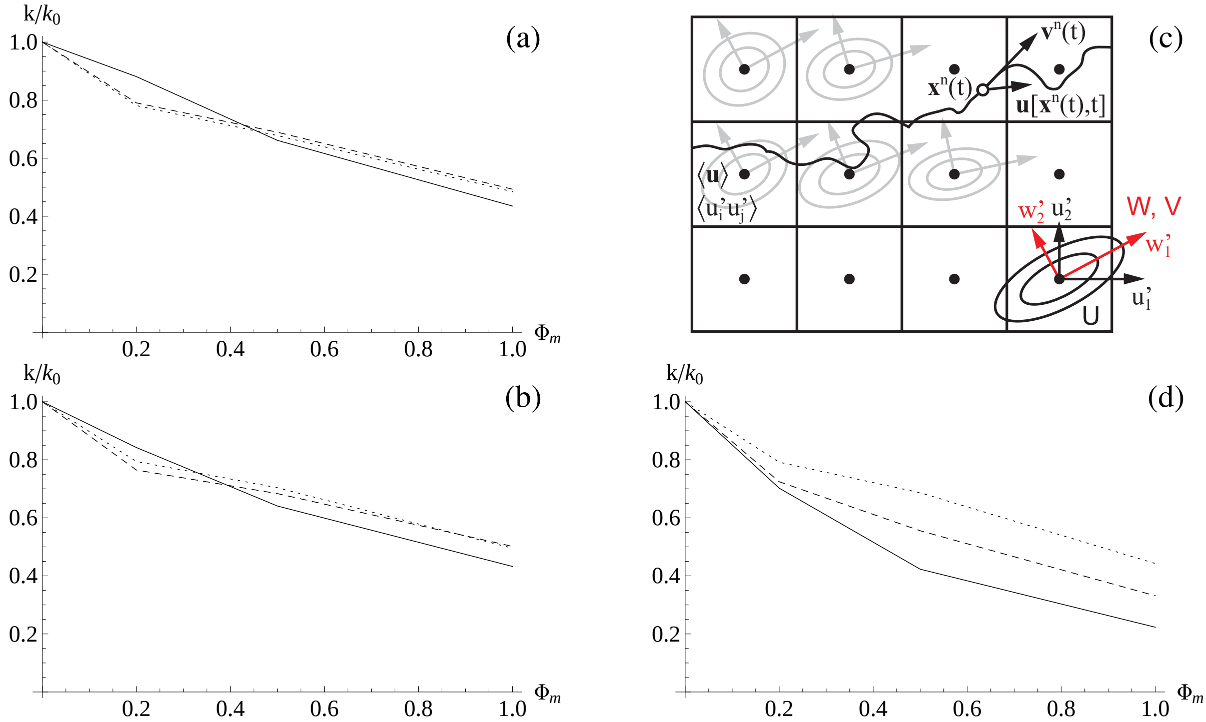

Figure 1. Attenuation of turbulent kinetic energy k as predicted by (a) the DNS' [2] and (b) the model for different mass loadings Φ_m and particle diameters, (solid) $\tau_p = 0.069$s; (dashed) $\tau_p = 0.251$s; (dotted) $\tau_p = 0.698$s. To enforce consistency of the seen velocity statistics, the Reynolds stress diagonalization depicted in (c) is applied as a particle travels through the computational grid of the RANS solution. In panel (d), model predictions are provided, where preferential concentration effects were neglected in the model for T_L^*.

By evolving an ensemble of particles with equations (1) and (2), the drag forces $\mathbf{F}^n$ of different particles can be evaluated in each RANS solution grid cell and the particle-phase terms in the fluid-phase RANS and Reynolds stress equations can be calculated. Unlike the model of Minier et al. [8], our new model enforces consistency of seen and fluid-phase velocity statistics and accounts for preferential concentration effects that are important for particles with St ≈ 1 as is shown next.

RESULTS

To verify the new particle-phase model, we compare model predictions with the detailed direct numerical simulation (DNS) study of Boivin et al. [2]. In their DNS', particles with different response times τ_p were suspended in forced homogeneous isotropic turbulence at different mass loadings Φ_m. The DNS attenuation results of the turbulent kinetic energy k compared to the particle-free case with kinetic energy k_0 are provided in Figure 1(a). The results from the model depicted in Figure 1(b) are in very good agreement with the DNS results for all particle types and loadings. In the model calculations plotted in Figure 1(d), preferential concentration effects are ignored in $T_L^*(\mathrm{St})$, which leads especially for particles with small τ_p and St $\approx 1 \ldots 5$ to an over prediction of turbulence attenuation.

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