A k-ε-$\overline{v^2}$ -f MODEL FOR TURBULENT FLOW OF DILUTE POLYMER SOLUTIONS UP TO THE MAXIMUM DRAG REDUCTION

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Abstract A k-ε-$\overline{v^2}$ -f model is developed to model turbulent flow of dilute polymer solutions up to the maximum drag reduction limit, by utilizing the Finitely Extensible Nonlinear Elastic-Peterlin (FENE-P) rheological constitutive model. Eight sets of direct numerical simulation (DNS) data are used to analyze the budgets and the behavior of relevant physical quantities, such as the nonlinear terms in the FENE-P constitutive equation, the turbulent kinetic energy transport equation, the wall normal Reynolds stress transport equation and the solvent dissipation transport equation. Calculated polymer stress, velocity profiles and turbulent flow characteristics are all in good agreement with current, and independent DNS data over a wide range of rheological and flow conditions, and show significant improvements over the corresponding predictions of other existing turbulence models for FENE-P fluids.

Introduction

DNS simulation of turbulent viscoelastic flow is significantly more expensive than Newtonian DNS at the same Reynolds number. The CPU-time and memory requirements are at least one order of magnitude larger as compared to the Newtonian case, and it is not feasible for most of the engineering purposes. Hence, Reynolds-averaged Navier–Stokes (RANS) or other less demanding models have to be developed for modeling turbulent flows of dilute polymer solutions in engineering applications.

Iaccarino et al. [1] were the first to introduce a k-ε-$\overline{v^2}$ -f model for fully developed channel flow of viscoelastic fluids for the whole range of drag reduction (DR). However, their predictions of the polymer shear stress in the Reynolds-averaged momentum equation, and of the budgets of the turbulent kinetic energy and of the evolution equation for the conformation tensor are not in agreement with DNS results. In this work we aim to address these shortcomings by presenting a new set of closures for FENE-P fluids and test it over the whole range of drag reduction in fully-developed turbulent channel flow, which in our view is essential to a future extension to other flows.

Governing equations

In what follows, upper-case letters or overbars denote Reynolds-averaged quantities and lower-case letters or primes denote fluctuating quantities. A hat denotes an instantaneous quantity.
The time-averaged momentum equation appropriate for incompressible flow of FENE-P fluids is:

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \overline{P}}{\partial x_i}-\frac{\partial}{\partial x_k}(\overline{\rho u_i u_k})+\frac{\partial \overline{\tau}_{ik}}{\partial x_k}$$

where $\overline{\tau}_{ik}$ is the time-averaged extra stress tensor, U_i is the mean velocity, $\overline{P}$ is the mean pressure, ρ is the fluid density and $\overline{\rho u_i u_k}$ is the Reynolds stress tensor. The extra stress tensor $\overline{\tau}_{ij}$ is defined as the sum of Newtonian and polymeric contributions described by the FENE-P rheological constitutive equation as:

$$\overline{\tau}_{ij}=2\eta_s S_{ij}+\overline{\tau}_{ij,p}$$

where the time-averaged polymer stress and functions of the average trace of the conformation tensor are given by:

$$\overline{\tau}_{ij,p}=\frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij}-f(L)\delta_{ij}\right]+\frac{\eta_p}{\lambda}\overline{f(C_{kk}+c_{kk})c_{ij}}\qquad f(C_{kk})=\frac{L^2-3}{L^2-C_{kk}}\qquad f(L)=1$$

L^2 is the maximum extensibility of the dumbbell and λ is the relaxation time of the polymer.

$$\left(\frac{\partial C_{ij}}{\partial t}+U_k\frac{\partial C_{ij}}{\partial x_k}\underbrace{-C_{jk}\frac{\partial U_i}{\partial x_k}-C_{ik}\frac{\partial U_j}{\partial x_k}}_{M_{ij}}\right)+\left(\underbrace{\overline{u_k\frac{\partial c_{ij}}{\partial x_k}}}_{CT_{ij}}-\left(\underbrace{\overline{c_{jk}\frac{\partial u_i}{\partial x_k}}+\overline{c_{ik}\frac{\partial u_j}{\partial x_k}}}_{NLT_{ij}}\right)\right)=\underbrace{-\frac{1}{\lambda}\left[f(C_{kk})C_{ij}-f(L)\delta_{ij}+\overline{f(C_{kk}+c_{kk})c_{ij}}\right]}_{\overline{\tau}_{ij}/\eta_p}$$

Here M_{ij} is exact and does not need a closure. On the other hand NLT_{ij}, CT_{ij}, and the time averaged polymer stress are nonlinear terms and developing models for these terms are main tasks of this work.
For calculating the Reynolds stress we adopt Boussinesq's turbulent stress–strain relationship:

$$-\rho\overline{u_i u_j} = 2\rho\nu_T S_{ij} - \frac{2}{3}\rho k\delta_{ij}$$

where the eddy viscosity ν_T is modeled according to k-ε-$\overline{v^2}$-f model [2] as:

$$\nu_T = C_\mu \overline{v^2} T_t \qquad T_t = \max\left\{\frac{k}{\varepsilon^2}, 6\sqrt{\frac{\nu}{\varepsilon}}\right\}$$

The transport equations for k and for its dissipation rate by the Newtonian solvent are:

$$U_j\frac{\partial k}{\partial x_j} = P_{kk} - \varepsilon + \frac{\partial}{\partial x_j}\left(\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) - \underbrace{\left(\overline{\tau_{ij}^p \frac{\partial u_i}{\partial x_j}}\right)}_{\text{Viscoelastic stress work}} + \underbrace{\frac{\partial}{\partial xj}\left(\overline{\tau_{ij}^p u_i}\right)}_{\text{Viscoelastic turbulent transport}}$$

$$U_j\frac{\partial \varepsilon}{\partial x_j} = \frac{C_{\varepsilon 1}P_k - C_{\varepsilon 2}\varepsilon}{T_t} - \varepsilon + \frac{\partial}{\partial x_j}\left(\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial \varepsilon}{\partial x_j}\right) - \underbrace{E_p}_{\text{Viscoelastic contribution}}$$

The transport equation for the scalar $\overline{v^2}$, is modified for viscoelastic fluids as:

$$U_j\frac{\partial \overline{v^2}}{\partial x_j} = kf + \frac{\partial}{\partial x_j}\left(\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial \overline{v^2}}{\partial x_j}\right) - 6\frac{\varepsilon}{k}\overline{v^2} - \underbrace{\varepsilon_{p,yy}}_{\text{Wall normal viscoelastic Stress work}} + \underbrace{Q_{p,yy}}_{\text{Wall normal viscoelastic turbulent transport}}$$

The indicated viscoelastic terms on the right hand side of the k, $\overline{v^2}$ and ε equations need closures. The other remaining terms are Newtonian like terms, and are explained in [2]. The viscoelastic closures were developed after an extensive analysis of DNS data. Figures (1) and (2) compare predictions of turbulent kinetic energy, wall normal turbulent fluctuations, and the trace of the polymer stress tensor at $\mathrm{Re}_{\tau 0} = 395$, with DNS data for L^2=14400 and a Weissenberg number of $We_{\tau 0} = 100$ corresponding to 37% drag reduction.

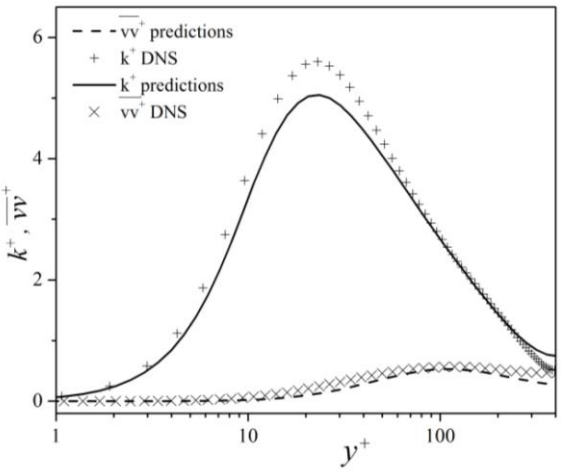

Fig. (1) Comparison between predicted and DNS data for k and $\overline{v^2}$

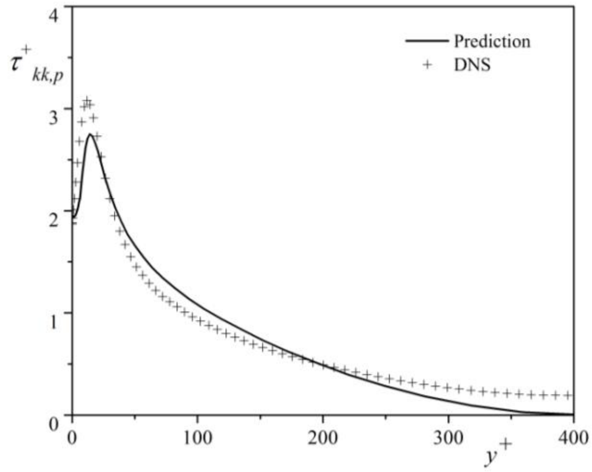

Fig. (2) Comparison between predicted and DNS data for the trace of the polymer stress tensor

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