ENERGY SPECTRA AND CHARACTERISTIC SCALES OF QUANTUM TURBULENCE INVESTIGATED BY NUMERICAL SIMULATIONS OF THE TWO-FLUID MODEL

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Abstract Quantum turbulence at finite temperature (within the framework of the two-fluid model) exibits an "anormal" distribution of kinetic energy of its superfluid component at scales larger than the inter-vortex distance. This anormal behavior is consistent with a thermalization of superfluid excitations at small scales. An original phenomenological argument allows us to predict explicitly the extension of the thermalization range. It is predicted that this extension is independent of the Reynolds number, and scales as the inverse square root of the normal fluid fraction. The prediction is well supported by high-resolution pseudo-spectral simulations of the two fluid-model.

INTRODUCTION

At finite temperature below $T_\lambda \approx 2.17$K, liquid helium (He-II) exhibits superfluid properties and may be described as the superposition of two interacting fluids: a normal fluid that has non-zero viscosity, and an inviscid superfluid in which vorticity is confined to quantized vortices. The macroscopic dynamics of this two-fluid system obeys respectively the Navier-Stokes and Euler equations (at low Mach number) coupled together by a mutual friction force which encompasses the interactions between the quantized vortices and the thermal excitations associated with the normal-fluid component. This is the general framework of the two-fluid model initiated by Landau and Tisza [1]. This is a continuous description of quantum-fluid dynamics which is expected to hold at lenght scales larger than the typical distance between quantized vortices.

Like in classical fluid, turbulent dynamics can be generated in quantum fluids and it is generally referred to as quantum turbulence in the literature [2]. Since Vinen's pioneering experiments about half a century ago, turbulence in quantum fluids (such as Bose-Einstein condensates, superfluid helium or neutron stars) has attracted much attention, and over the last two decades, a central motivation has been to examine to what extent quantum turbulence resembles, or differs, from classical turbulence.

In classical turbulence, a cascade of kinetic energy operates from the large scales to the small scales, at which energy is eventually dissipated by the viscosity. This dynamical state is characterized by a $k^{-5/3}$ distribution of kinetic energy among wavenumbers with an intrinsic (ultra-violet) cut-off at $k_d \sim \nu^{-3/4}$ imposed by the viscosity of the fluid according to the Kolmogorov's theory. In quantum turbulence at finite (non-vanishing) temperature, the situation is different since the superfluid component is inviscid and, therefore, can not dissipate kinetic energy thanks to the viscosity. Energy dissipation in the superfluid component can only occur through the mutual friction with the normal-fluid component. Let us mention that in this picture, dissipation processes related to quantized-vortex reconnection is neglected since the temperature is supposed to be "sufficiently larger than zero" so that dissipation processes at larger scales prevail. Another major difference with classical turbulence is about the cut-off of small-scale superfluid fluctuations. Since the superfluid is inviscid, the viscous cut-off is here rejected to infinity. The relevant physical cut-off is given by the inter-vortex distance, δ, beyond which hydrodynamical excitations (e.g. Kelvin waves along superfluid vortices) are expected to be vanishingly small in the presence of normal fluid. This characteristic length-scale is related to the vorticity of the superfluid and the quantum of circulation, κ.

RESULTS

During the last four years, we have been involved in direct numerical simulations of the two-fluid model in the configuration of homogeneous and isotropic quantum turbulence [3, 4]. Interestingly, we have shown that the stationary distribution of kinetic energy for the superfluid component exhibits a $k^{-5/3}$ dependence at large scales, consistent with a cascade of kinetic energy towards small scales, but also a dependence close to k^2 at scales larger than the inter-vortex distance (see Figure). This "anormal" distribution of energy near the inter-vortex distance is consistent with a thermalization of superfluid excitations, that reenforces the mutual friction with the normal-fluid component and allows the superfluid to dissipate at small scales the kinetic energy fueled the large-scale energy cascade. The balance between this two behaviors defines a new characteristic mesoscale, ℓ, that separates the energy cascade and the thermalization process.

During this ETC14 conference, we propose to present the general picture of quantum turbulence (at finite temperature) within the framework of the two-fluid model, with a particular focus on the "anormal" distribution of kinetic energy of the superfluid component. We will show that an original phenomenological argument allows us to predict explicitly the ratio δ/ℓ characterizing the extension of the range of thermalization of the superfluid fluctuations. In particular we will

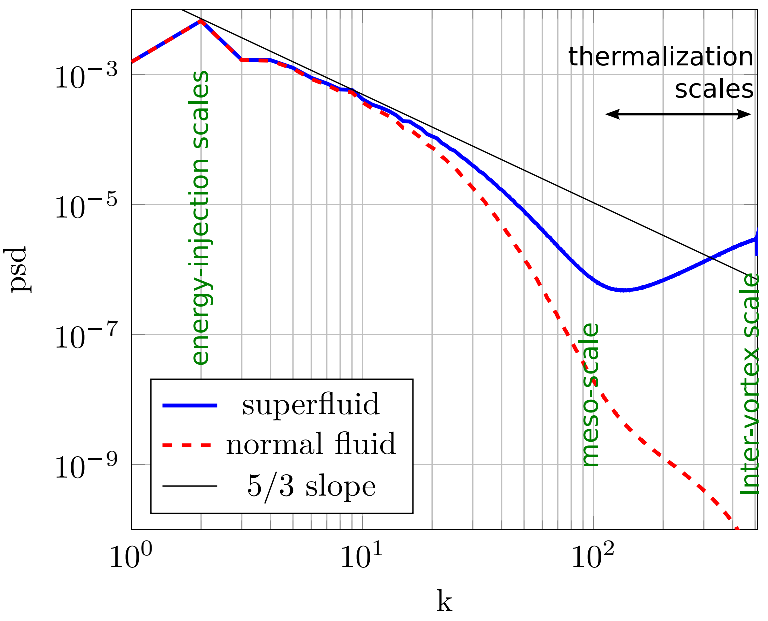

Figure 1. Energy distribution of the superfluid component exhibits an “anormal” range of thermalized scales. This range extends typically from the characteristic mesoscale ℓ to the inter-vortex distance δ. Simulations at $T \simeq 1.15\,K$.

show that this extension is independent of the Reynolds number, and scales as the inverse square root of the normal fluid fraction. This prediction is well supported by high-resolution pseudo-spectral simulations of the two fluid-model.

References

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