CAMASSA-HOLM TYPE EQUATIONS AND VORTEXONS IN AXISYMMETRIC POISEUILLE PIPE FLOWS

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Abstract We present a study on the nonlinear dynamics of a disturbance to the laminar state in non-rotating axisymmetric Poiseuille pipe flows. We show that the associated Navier-Stokes equations can be reduced to a set of coupled Camassa-Holm type equations. These support regular and singular inviscid travelling waves with wedge-type singularities, the so called peakons, which bifurcate from smooth solitary waves as their celerity increase. In physical space they correspond to localized/periodic toroidal vortices concentrated near the pipe boundaries (wall vortexon) or that wrap around the pipe axis (centre vortexon). The dynamics of a vortexon is also investigated by means of an accurate Fourier-based numerical scheme.

Camassa-Holm type equations for axisymmetric pipe flows

Recently, in [1] the dynamics of non-rotating axisymmetric pipe flows is studied in terms of solitons and travelling waves of nonlinear wave equations. At high Reynolds numbers, the dynamics of small but finite long-wave perturbations of the laminar flow obey a coupled system of nonlinear Korteweg-de Vries-type (KdV) equations that generalize the one-component KdV model derived in [2] to study propagation of waves along the core of concentrated vortex flows. The coupled KdV equations support inviscid soliton and periodic wave solutions in the form of toroidal vortex tubes, hereafter referred to as vortexons, which are similar to the inviscid nonlinear neutral centre modes found by [3]. These vortical structures eventually slowly decay due to viscous dissipation on the time scale $t \sim O(Re^{6.25})$ (see [1]). Here, we extend the analysis in [1] and show that the axisymmetric Navier-Stokes equations for non-rotating pipe flows can be reduced to a set of coupled Camassa-Holm (CH) type equations [4,5].

Vortexons

For the coupled CH equations, regular traveling waves are numerically computed by means of the Petviashvili method (see, for example, [6-7]). In physical space they correspond to localized/periodic toroidal vortices concentrated near the pipe boundaries (wall vortexon) or that wrap around the pipe axis (centre vortexon). Singular vortexons with discontinuous radial velocities are also numerically identified as associated to special traveling waves with wedge-type singularities, viz. peakons and shocks [8]. They bifurcate from smooth solitary waves as their celerity increase.
Dissipation rules out the existence of such singular perturbation flows. Indeed, we observe that an initial vortical patch splits into a stable centre vortexon radiating patches of vorticity near the wall. These can undergo further splitting before the viscous time scale, leading to a proliferation of centre vortexons (see Fig. 1 and 2) similarly to the puff splitting in slug flows. The splitting process originates from radial fluxes of azimuthal vorticity from the wall to the pipe axis in agreement with the inverse cascade of cross-stream vorticity identified in [9]. The inviscid vortexon is similar to the nonlinear neutral structures derived in [3] and it may be a precursor to puffs and slugs observed at transition, since most likely it is unstable to non-axisymmetric disturbances.

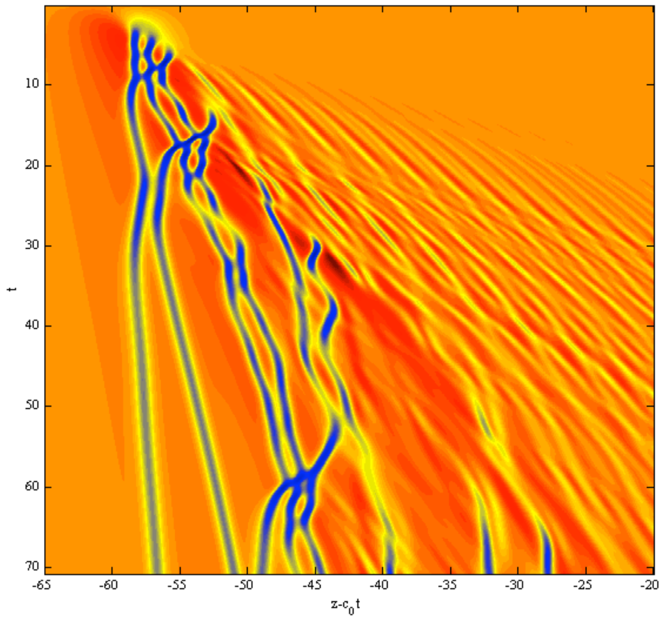

Figure 1. Space-time plot of the trace of traveling waves associated to vortexons.

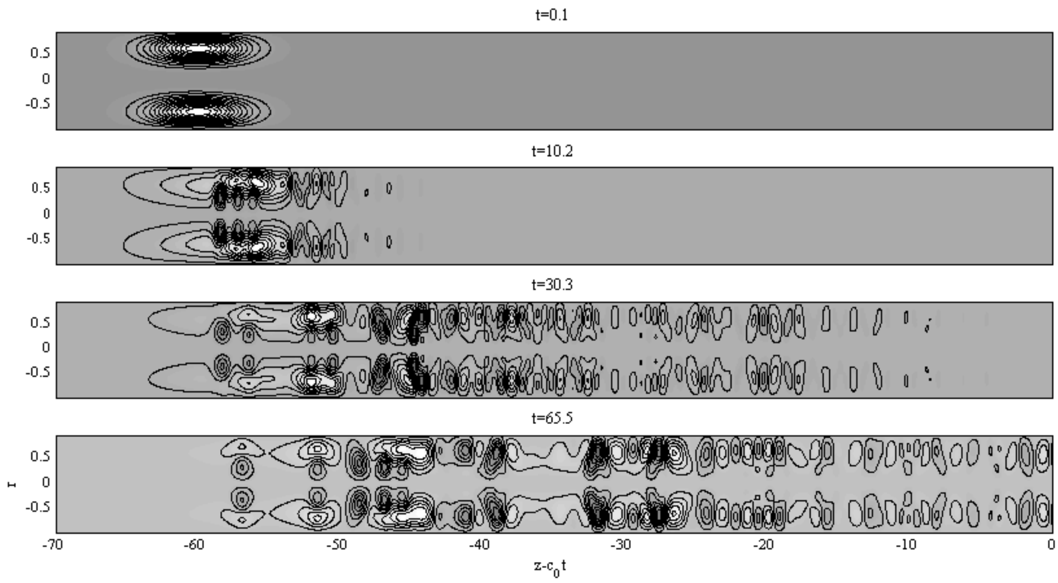

Figure 2. Evolution of a perturbation according to the Camassa-Holm equations for axisymmetric flows.

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