TEMPERATURE AND VELOCITY GRADIENTS IN TURBULENT CONVECTION

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Abstract High-resolution direct numerical simulations based on the finite difference and spectral element methods are used to study the fine-structure of convective turbulence in different regions of the convection cell. Similar to previous studies in homogeneous box turbulence and simple shear flows, it turns out that the resolution requirements are then much more demanding in comparison to the standard grids used to study large-scale features of turbulence. We investigate the distributions of the locally varying dissipation and diffusion scales which are connected with high-amplitude events of the dissipation rate fields.

MOTIVATION

Many turbulent flows in nature and technology are driven by sustained temperature differences. Applications range from cooling devices of chips to convection in the Earth and the Sun. Turbulent Rayleigh-Bénard convection (RBC) is the paradigm for all these convective phenomena because it can be studied in a controlled manner, but it still has enough complexity to contain the key features of convective turbulence in the examples just mentioned. RBC in cylindrical cells has been studied intensely over the last few years in several laboratory experiments, mostly in slender cells of aspect ratio smaller than or equal to unity in order to reach the largest possible Rayleigh numbers. The key question in RBC is the mechanism of turbulent transport of heat and momentum (see e.g. [1] for a recent review). Since the fluid motion is driven by a constant temperature difference between the top and bottom plates, thin boundary layers of temperature and velocity will form on these walls as well as on the side walls of the cell. A deeper understanding of the global transport mechanisms is possible only if we understand the dynamical coupling between the boundary layers and the rest of the flow in the bulk of the cell. The majority of the direct numerical simulation (DNS) studies of the last years have been focussed on the dynamics in the boundary layers which turned out to be fully three-dimensional and time-dependent and which is tightly coupled to the large-scale circulation (see e.g. Refs [5, 7, 8]). In the present work, we will focus on the properties of the small-scale turbulence which is tightly coupled to the gradients of temperature and velocity fields and thus both dissipation rate fields, the thermal dissipation rate $\epsilon_T(\mathbf{x}, t)$ and the kinetic energy dissipation rate $\epsilon(\mathbf{x}, t)$. Both fields are known to be highly intermittent in any fully developed turbulent flow [2]. Resolving their statistics properly is demanding in terms of the grid resolution as demonstrated in homogeneous isotropic turbulence [6] or in plane shear flows [3]. High spectral accuracy and finite-sized cell geometry are combined in a spectral element method (SEM) which is used for the present studies [4] and compared with a finite difference method (see [7] for references).

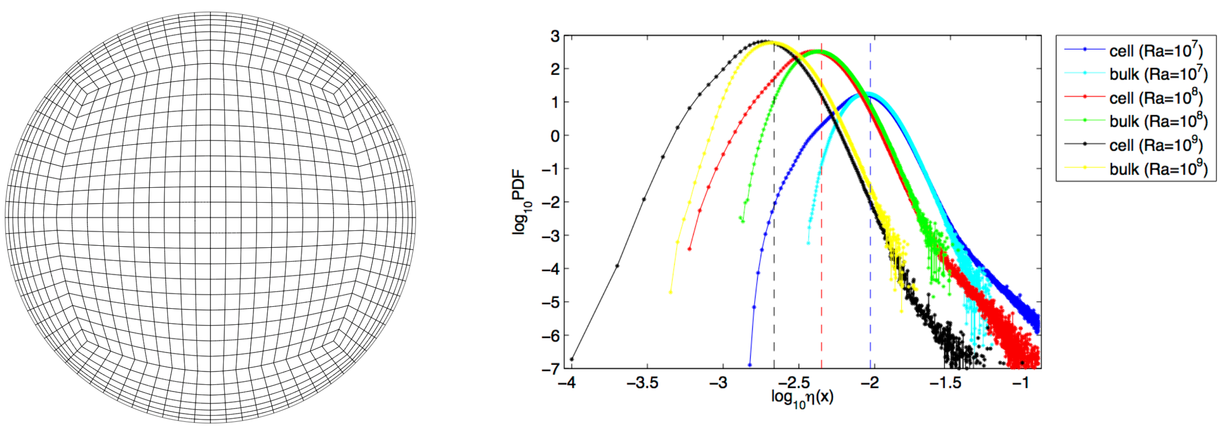

Figure 1. Left: Horizontal element mesh. Right: Distribution of local dissipation scales.

NUMERICAL EXPERIMENTS

We solve the three-dimensional Boussinesq equations numerically. The height of the cell H, the free-fall velocity $U_f = \sqrt{g\alpha\Delta TH}$ and the imposed temperature difference ΔT are used to rescale the equations of motion. The three control parameters of Rayleigh-Bénard convection which result are the Rayleigh number Ra, the Prandtl number Pr and the aspect ratio $\Gamma = D/H$ with the diameter D. This results in the following dimensionless form of the equations of motion

$$\tilde{\boldsymbol{\nabla}} \cdot \tilde{\mathbf{u}} = 0\,, \tag{1}$$

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\boldsymbol{\nabla}})\tilde{\mathbf{u}} = -\tilde{\boldsymbol{\nabla}}\tilde{p} + \sqrt{\frac{Pr}{Ra}}\tilde{\boldsymbol{\nabla}}^2\tilde{\mathbf{u}} + \tilde{T}\mathbf{e}_z\,, \tag{2}$$

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\boldsymbol{\nabla}})\tilde{T} = \frac{1}{\sqrt{RaPr}}\tilde{\boldsymbol{\nabla}}^2\tilde{T}\,, \tag{3}$$

where

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa}\,, \qquad Pr = \frac{\nu}{\kappa}\,, \tag{4}$$

with gravity acceleration g, thermal expansion coefficient α, kinematic viscosity ν, and thermal diffusivity κ. Throughout the study we set $\Gamma = 1$. At all walls no-slip boundary conditions for the fluid are applied, $\mathbf{u} = 0$. The side walls are adiabatic, i.e., the normal derivative of the temperature field vanishes, $\partial T/\partial \mathbf{n} = 0$. The top and bottom plates are held at a fixed temperatures $T = 0$ and 1. In response to the input parameters Ra, Pr and Γ a turbulent heat flux from the bottom to the top plate is established. The presented simulations are done for Prandtl numbers $Pr = 0.021, 0.7$ and 6 and for Rayleigh numbers $Ra = 10^7, 10^8$ and 10^9. Spectral element meshes (see Fig. 1 left for an example) with up to 875000 elements and polynomial orders on each element up to $N = 11$ make the DNS at the moderate Rayleigh numbers quite demanding and require massively parallel computations on up to 16384 cores.

In the right panel of Fig. 1 we present one result of our ongoing studies. The probability density functions (PDF) of the local dissipation scales for $Pr = 0.7$, $Ra = 10^7$, 10^8, 10^9 and a polynomial order $N = 7$ are displayed. Such local dissipation scales generalize the classical mean Kolmogorov scale which is based on the mean energy dissipation rate by incorporating the local fluctuations of the dissipation rate field. They are defined here as (for alternative definitions see Ref. [3])

$$\eta(\mathbf{x}, t) = \frac{\nu^{3/4}}{\epsilon(\mathbf{x}, t)^{1/4}}\,. \tag{5}$$

We analyze the PDFs obtained in the whole convection cell with those obtained in the bulk which is defined as the sub volume $V_b = \{\tilde{\mathbf{x}} \mid 0 \leq \tilde{r} \leq 0.3; 0.2 \leq \tilde{z} \leq 0.8\}$. The dashed lines are displayed in the same colors as the corresponding data sets and show the mean Kolmogorov scale $\langle \tilde{\eta}_K \rangle = [Pr^2/(Nu-1)Ra]^{1/4}$. For consistency, we also checked that $\langle \tilde{\eta}_K \rangle$ agrees very well with the mean scale that is directly evaluated from the PDF. It can be seen that the fluctuating scales are distributed around the mean dissipation scale. When the boundary layers are included smaller and larger local dissipation scales are obtained. This is in agreement with the higher amplitudes of both dissipation rates in the boundary layers. Experimentally, these scales have been investigated recently by Zhou and Xia [9] for convection in water.

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