# FINITE REYNOLDS NUMBER EFFECTS ON PRESSURE IN FREELY DECAYING ISOTROPIC TURBULENCE

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*Abstract* The finite Reynolds number (FRN) effects over the statistics of the pressure spectrum in free decaying isotropic turbulence are investigated by the eddy-damped quasi-normal Markovian (EDQNM) model. The analysis of the results leads to the conclusion that these effects are important even at high Reynolds numbers, for which an asymptotic behaviour is usually assumed. The results also suggest that $Re_\lambda = O(10^4)$ must be considered to observe a one-decade inertial range in the pressure spectrum with Kolmogorov $-7/3$ scaling. This threshold value is larger than almost all existing DNS and experimental data, justifying the discussions about other possible scaling laws. The -5/3 slope reported in some DNS data is recovered as well by the EDQNM model, but it is observed to be a low-Reynolds effect.

## INTRODUCTION

Since the seminal works by Taylor[1], a number of important theoretical insights about the emergence of self-similar regimes in homogeneous isotropic turbulence (HIT) free decay has been reported in open literature[2, 3, 4]. The theoretically predicted HIT decay behaviour has not always been observed in experiments and numerical simulations, due to a number of factors that influence realistic investigations. Indeed, experiments are affected by non reducible epistemic uncertainties, such as the precision of the measurements and the limited physical domain investigated. On the other hand, the numerical simulation of HIT decay is driven by the initial conditions prescribed, which implies that a spontaneous emergence of robust decay regimes cannot be captured by a single run. The physical properties of the turbulent flows play an important role as well. A number of different effects, such as intermittency or the finite Reynolds number, are usually advocated when a deviation of the results from the theoretical expected behaviour is observed.
The present work aims to clarify this issue, giving an accurate quantification of the FRN effects on HIT free decay. The EDQNM model is used to perform this analysis. This model, which is based on the discretisation of the Lin equation:

$$\frac{\partial E(k,t)}{\partial t} + 2\nu k^2 E(k,t) = T(k,t) \tag{1}$$

where $k$ is a spectral element, $t$ the characteristic time, $\nu$ the kinematic viscosity, $E(k,t)$ the energy spectrum and $T(k,t)$ the non linear energy transfer, is an excellent candidate to this purpose due to its accuracy in the prediction of HIT statistical quantities[5, 6]. Moreover, it does not naturally take into account the effects of intermittency, so that FRN effects can be isolated.
A particular attention has been devoted to the analysis of the pressure spectrum statistics. Sets of EDQNM simulations have been performed to investigate the FRN effects, considering values of the slope of the energy spectrum at the large scale $\sigma$ in the interval $\sigma \in [1,\, 4]$. The results have been analysed in the range $10^{-3} \leq Re_\lambda \leq 10^6$.

## ANALYSIS OF THE PRESSURE SPECTRUM STATISTICAL QUANTITIES

Let us first consider the compensated pressure spectrum $E_p/(\varepsilon^{4/3}k^{-7/3})$ shown in Figure 1 (a). At very high $Re_\lambda$, a plateau in the inertial range is observed for three decades. A bump in the compensated pressure spectrum is also observed at scales close to the dissipation region. This phenomena is classically referred to as *bottleneck effect* when dealing with the energy spectrum. The slope of this bump, computed by EDQNM results, is about 1/4 at very high Reynolds numbers and progressively increases up to 3/10 for $Re_\lambda \approx 600$. This behaviour can be observed in Figure 1 (b), were the local slope of the pressure spectra is reported. For pressure spectra at progressively decreasing $Re_\lambda$, the plateau becomes less and less visible, and the bump region degenerates into a small secondary plateau whose slope is $\approx -5/3$.
The investigation of the pressure related statistics regards the underlying relation between the decay of the Taylor microscale $\lambda$ and the equivalent scale for the pressure spectrum $\lambda_p^2 = \rho^2\left(u^2\right)^2/\left(\nabla p\right)^2$, where $\rho$ is the flow density and $\left(\nabla p\right)^2$ is the turbulent fluctuation of the pressure gradient. In the early works of Batchelor[7], which relied on the joint Gaussian assumption (JGA) hypothesis, the following relations have been derived in the case of high and low $Re_\lambda$:

$$\begin{cases} \lambda_p/\lambda & = 0.11 Re_\lambda^{1/2}\,, \quad Re_\lambda \to \infty \\ \lambda_p/\lambda & = 0.81\,, \qquad\qquad Re_\lambda \to 0 \end{cases} \tag{2}$$

EDQNM results (see Figure 2) show that the curves $\lambda_p/\lambda$ for Saffman and Batchelor turbulence converge toward a very similar value, which is approximately $\lambda_p/\lambda = 0.62$, for low $Re_\lambda$. Moreover, the behaviour at high $Re_\lambda$ is compatible

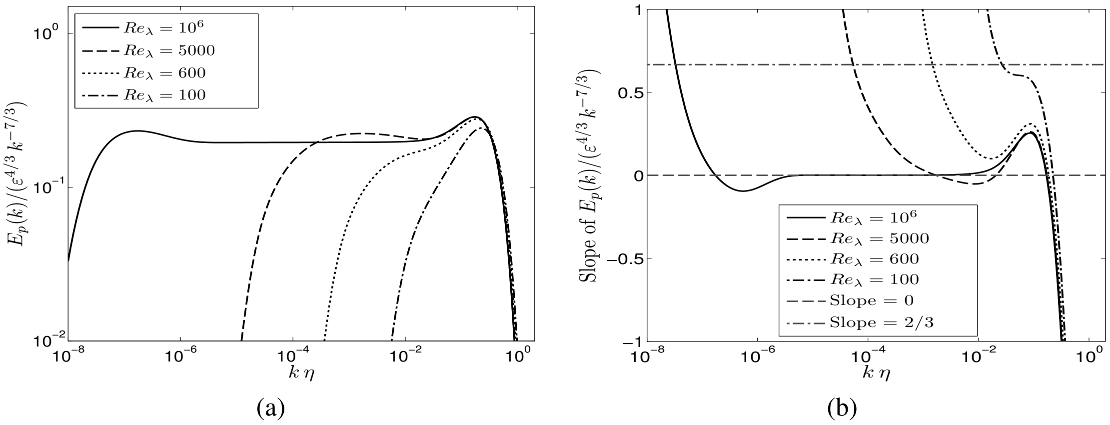


**Figure 1.** (a) Compensated pressure spectrum $E_p = E_p/(\varepsilon^{4/3} k^{7/3})$ and (b) local slope of the compensated pressure spectrum in the case of Saffman turbulence.

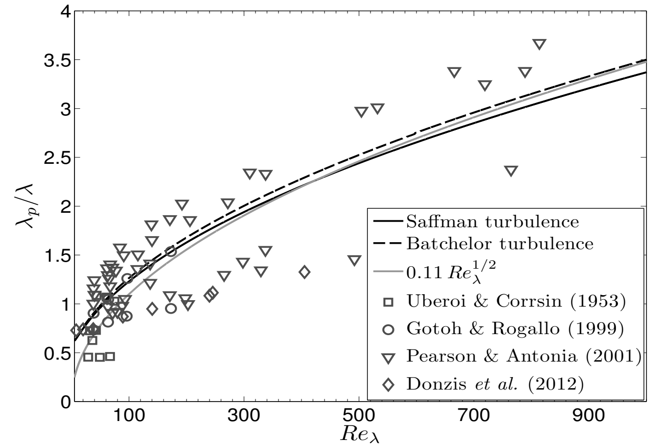


**Figure 2.** Ratio of the Taylor microscales $\lambda_p/\lambda$ for moderate to low $Re_\lambda$, compared with experiments and DNS results.

with the relation $\lambda_p/\lambda = 0.11 Re_\lambda^{1/2}$. This result is in qualitative agreement with Batchelor[7] and in very good agreement with the experimental results[8, 9]. Several DNS studies in literature have observed a departure from the JGA behaviour $\lambda_p/\lambda = 0.11 Re_\lambda^m$, $m = 0.5$, which is usually credited to intermittency. Donzis et al.[10] reported a better fit power law of $m = 0.34$. The EDQNM results point out that the values observed in the cited DNS study are matched in the range $30 \leq Re_\lambda \leq 100$. The results, which are discussed in depth in the work by Meldi & Sagaut[11], lead to the conclusion that FRN effects over pressure statistics are not negligible for $Re_\lambda \leq O(10^4)$.

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