NEW MEAN VELOCITY SCALING LAWS FOR TURBULENT POISEUILLE FLOW WITH WALL TRANSPIRATION

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Abstract A new mean velocity scaling law for a turbulent Poiseuille flow with wall transpiration was found using Lie group analysis and validated thereafter using DNS. The region of validity covers the whole core region of the channel. Though the scaling law is of log type the slope constant differs from the von Kármán constant and is equal to 0.3. Further, extended forms of the linear viscous sublayer law and the near-wall log-law have been derived, which, as a particular case, include the laws related to the wall-bounded flows without transpiration.

NEW MEAN VELOCITY SCALING LAW

A fully developed, plane constant-mass-flux, turbulent Poiseuille flow with wall transpiration, i.e. uniform blowing and suction velocity v_0 on the lower and upper walls correspondingly, is investigated by direct numerical simulation (DNS) of the three-dimensional, incompressible Navier-Stokes equations. The DNS is conducted at $Re_\tau = 250, 480$ for different relative transpiration velocities ($v_0/u_\tau = 0.05, 0.1, 0.16, 0.26$). The friction velocity is defined as $u_\tau = \sqrt{(u_{\tau b}^2 + u_{\tau s}^2)/2}$. A numerical code developed by the School of Aeronautics, Technical University of Madrid ([1]) is used for the DNS.

One of the key objectives of the study is to further develop and validate Lie symmetry group theory for turbulent Poiseuille flow with wall transpiration at moderate and high Reynolds numbers. Lie symmetry analysis is used to find symmetry transformations and in turn invariant (self-similar) solutions of the set of two- and multi-point correlation (MPC) equations. Employment of the MPC equations for the velocity and pressure fluctuations together with corresponding continuity equations and some side conditions for MPC tensor facilitates generation of a complete statistical description of turbulence. Application of the set of scaling transformation Lie symmetry groups allows to construct exact solutions (scaling laws) to this set of MPC equations ([2], [3]). The observation that the transpiration is symmetry breaking led to the immediate conclusion that a new logarithmic deficit-type scaling law should be found in the center of the channel. Since Lie symmetry analysis allows us to construct only the functional form of the proposed new logarithmic scaling law:

$$\bar{U}_1 = A ln\left(\frac{x_2}{h} + B\right) + C \tag{1}$$

further determination of A, B and C is performed using results from DNS. It is found that new log-law has the following form:

$$\frac{\bar{U}_1 - U_b}{u_\tau} = \frac{1}{\gamma} ln\left(\frac{x_2}{h}\right) \tag{2}$$

while the value of the slope constant $\gamma \simeq 0.3$ is rather different from Kármán's constant. The new scaling law covers the core region of the flow and with increasing of the transpiration velocity v_0 the region of validity increases up to 75% of the channel hight (see figure 1a,b).

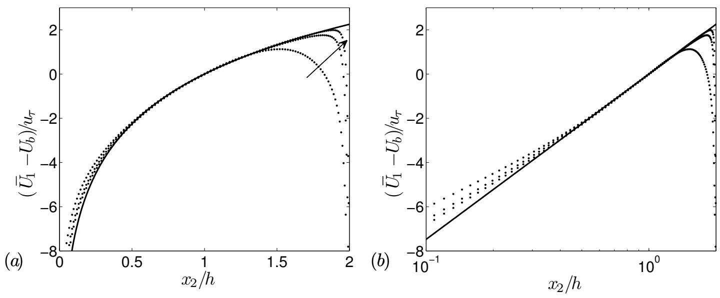

Figure 1. Mean velocity profiles in a) linear and b) semi-log scaling. The new log-law (eq. (2)): —— ($\gamma = 0.3$); $\bar{U}_1$: ······. In direction of arrow: $v_0/u_\tau = 0.05, 0.1, 0.16,\ Re_\tau = 480$.

NEAR-WALL MEAN VELOCITY SCALING LAWS

The occurrence of the convective momentum transport term on the left hand side of the mean momentum equation

$$v_0 \frac{d\bar{U}_1}{dx_2} = -\frac{d\bar{P}}{dx_1} - \frac{d\overline{u_1 u_2}}{dx_2} + \nu \frac{d^2\bar{U}_1}{dx_2^2} \tag{3}$$

modifies classical scaling laws (viscous sublayer and law of the wall) which were regarded as universal for all non-transpiring wall-bounded flows. While for moderate blowing/suction rates $0.05 < v_0/u_\tau < 0.1$ ([4], [5]) the viscous sublayer appears on both walls, near-wall log-law of the wall is observed only on the blowing side, where local friction velocity is considerably lower then on the suction side. The near-blowing-wall log-law has the following form:

$$\bar{U}_1^+ = \frac{1}{\kappa} ln\left(x_2^+\right) + C(v_0^+), \tag{4}$$

where $C(v_0^+)$ is an additive universal function which depends on transpiration rate, and reduces to the classical additive log-law constant C_0 and κ is von Kármán's constant. The new terms emphasize that transpiration changes only the additive constant ([6]). Since suction strongly suppresses turbulence, there is no log-law near the suction wall. DNS verification of the near-wall log-law scaling results are presented on the figure 2a,b.
For the viscous sublayers on blowing and suction sides respectively the following velocity scaling laws are obtained:

$$\bar{U}_1^+ = \frac{a}{h} x_2^+, \tag{5}$$

$$\bar{U}_1^+ = \frac{2-a}{h} x_2^+. \tag{6}$$

Here a is the constant that represents the distance from the blowing wall to the point of zero shear stress. Scaling for the viscous sublayer on the blowing/suction side is shown on the figure 3. It can be seen that the thickness of the viscous sublayer decreases as the transpiration rate increases.

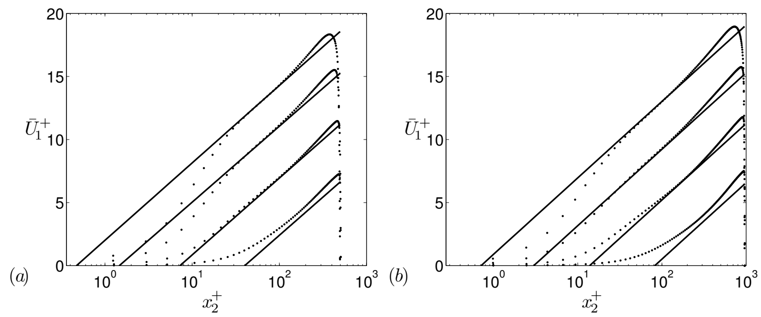

Figure 2. Mean velocity profiles of a turbulent channel flow with wall transpiration at the blowing wall at $a)$ $Re_\tau = 250$ and $b)$ $Re_\tau = 480$. The classical log law (eq. 4)): ——; $\bar{U}_1$: ······. From top to the bottom $v_0/u_\tau = 0.05, 0.1, 0.16, 0.26$.

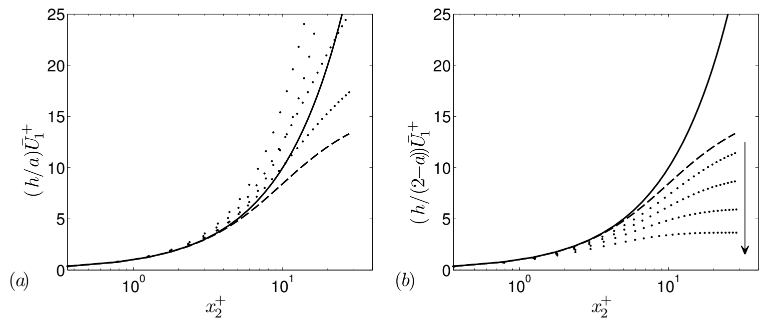

Figure 3. Mean velocity profiles and linear law. $a)$ at the blowing wall, $b)$ at the suction wall. The linear laws (eq. (5, 6)): ——; $\bar{U}_1$ without transpiration: - - ; $\bar{U}_1$ with transpiration: ······ . In direction of arrow: $v_0/u_\tau = 0.05, 0.1, 0.16, 0.26,\ Re_\tau = 250$.

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