TRANSITION TO TURBULENCE IN OSCILLATORY SUPERFLOWS

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Abstract The flow of superfluid helium-4 at very low temperatures around an oscillating microsphere has been studied in detail. At small oscillation amplitudes the drag force is linear in velocity amplitude. Above a critical velocity amplitude v_c a transition from potential flow to turbulent flow is signaled by a large and nonlinear drag force that scales as $(v^2 - v_c^2)$, where v_c is found to be independent of temperature (below 1 K) and of driving force. Interestingly, the critical velocity is found to scale as $v_c \sim \sqrt{\kappa\omega}$, where $\kappa = h/m \approx 10^{-7}$ m^2/s (h is Planck's constant and m is the mass of a helium-4 atom) is the circulation quantum of the superfluid vortices and $\omega/2\pi$ is the oscillation frequency ranging from about 120 Hz to 700 Hz. We also observe slightly above v_c that the flow pattern is unstable and switches intermittently between potential flow and turbulence. From time series recorded at constant drive and temperature we have investigated the statistical properties of this switching phenomenon. In this talk the universal law $v_c \sim \sqrt{\kappa\omega}$ will be discussed in detail. It can be derived theoretically in various ways: firstly, from a qualitative but very general argument based on the "superfluid Reynolds number" $R_s = vl/\kappa$ where v is the flow velocity and l is a characteristic length scale, secondly, and in more detail, from Kopnin's equation of the time dependence of the vortex line density, and, finally, we have obtained this law rigorously by dynamical scaling of the equations of motion of vortex dynamics.

THE EXPERIMENTAL SETUP

Figure 1 shows a schematic of our measuring technique [1, 2]. A magnetic microsphere (radius = 0.12 mm) is levitating inside of a parallel plate capacitor made of superconducting Nb electrodes (spacing d = 1 mm). It is carrying an electric charge q of about 1 pC because a voltage of several 100 V was applied to the capacitor before cooling through the superconducting transition temperature of Nb at about 10 K. Due to flux trapped in the electrodes the sphere levitates in a potential well and vertical oscillations can be excited by applying a small ac voltage U_{ac} to the electrodes at resonance frequency. The amount of trapped flux, and hence the resonance frequency, depend on the speed of the cool-down procedure. The velocity amplitude v of the charged sphere can be detected by measuring the displacement current $I = qv/d$. After testing the measuring cell in vacuum the capacitor is filled with liquid helium. At constant temperatures from 20 mK up to 1.9 K we measure $I(U_{ac})$ and plot $v = Id/q$ as a function of the driving force $F = qU_{ac}/d$.

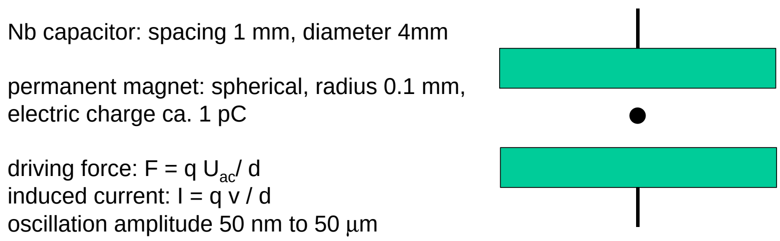

Figure 1. Sketch of the experimental technique.

THE FLOW REGIMES

Figure 2 shows the velocity amplitude v of the oscillating sphere as a function of the driving force F at a temperature of 0.3 K [2]. At low velocities the linear increase is in the regime of potential flow with a drag force that is linear in velocity. A quantitative analysis of the slope shows that the drag force is given by ballistic phonon scattering that vanishes as T^4 because of the vanishing phonon density. At velocities larger than a critical velocity $v_c \approx 20$ mm/s we observe a strong and nonlinear drag. A fit to the data indicates a drag force that is given by $\gamma(v^2 - v_c^2)$ and $\gamma = c_D \rho \pi r^2/2$, where ρ is the density of the superfluid and r is the radius of the sphere. The drag coefficient $c_D = 0.36$ corresponds to the classical value of about 0.4. In contrast, in a classical liquid where the transition to turbulence is gradual without an abrupt critical velocity, the turbulent drag of a sphere is given by γv^2. Therefore, the effect of superfluidity is the constant shift by $F_0 = \gamma v_c^2$ on the force axis, see Fig. 2. Finally, slightly above v_c there is an unstable regime (shaded area in Fig.2) where neither potential flow nor turbulence are stable. Here the flow switches intermittently between both patterns. With increasing drive the lifetime of the turbulent phases grows rapidly until the lifetimes are longer than the measuring time

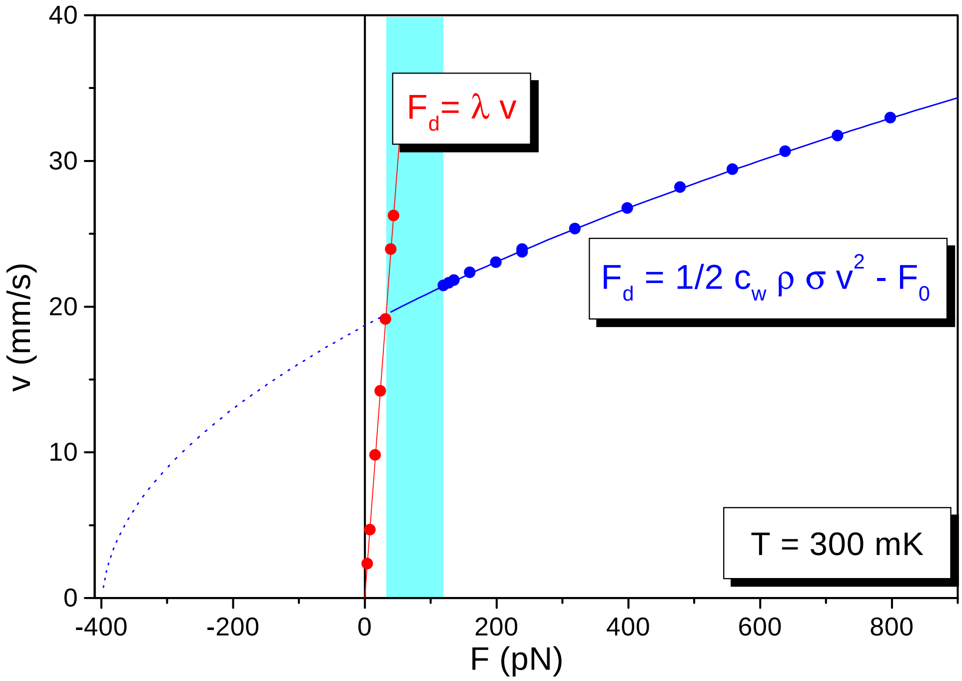

Figure 2. Velocity amplitude as a function of the driving force amplitude at 0.3 K.

and, hence, turbulence appears to be stable. This phenomenon has been analyzed in detail [3] and, because of limited space and time, will not be discussed here. Instead, we will concentrate on the critical velocity v_c.

THE CRITICAL VELOCITY FOR THE ONSET OF TURBULENCE

Experimentally we find that v_c is independent of the driving force and the temperature (below 1 K). There is, however, a frequency dependence scaling as $\sqrt{\omega}$ [4]. There are several possibilities to understand this result. A qualitative but very general argument is based on the "superfluid Reynolds number" as follows. In superfluid turbulence in the $T \to 0$ limit there is no viscosity. The classical Reynolds number is replaced by the "superfluid Reynolds number" by replacing the viscosity by the circulation quantum (note that both quantities have the same dimension m^2/s), i.e., $R_s = vl/\kappa$. It is generally assumed that turbulence may be expected when $R_s \geq 1$. For an oscillation amplitude $a = v/\omega$ that is much smaller than the size of the sphere, which is the case here, the characteristic length scale is $l = a$. Setting $R_s = 1$ at v_c we immediately obtain $v_c \sim \sqrt{\kappa\omega}$. This result is universal: only a numerical prefactor might have some geometry dependence. A more detailed approach is based on the dynamical properties of the superfluid vortex tangle [5]. It was shown by Kopnin [6] that the tangle has a relaxation time τ when responding to a change of the superfluid flow velocity v, namely (below 1 K) $\tau = 2\kappa/v^2$. The vortex tangle can respond to an oscillating flow field only if $\omega\tau \leq 1$. This gives $v_c \sim \sqrt{2\kappa\omega}$. A third method is based on the dynamical scaling of the equations of vortex motion [7]. Scaling the length by a factor λ, the times by λ^2 and, therefore, the velocities by $1/\lambda$, and taking into account that for oscillatory flow the frequency ω is a scaling variable, we find $v_c \propto \sqrt{\omega}$. This is a rigorous result. Therefore, our law $v_c \sim \sqrt{\kappa\omega}$ is well established.

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