CHARACTERIZATION OF LARGE SCALE QUANTITIES AND ENERGY SPECTRUM FOR VERY LARGE PRANDTL NUMBERS

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Abstract We investigate the scaling of the Nusselt and Péclet number as well as that of energy spectrum using direct numerical simulation for very large and ∞ Prandtl numbers. Simulations have been performed in a box for the Rayleigh numbers in the range $10^4 - 10^8$ and for the Prandtl numbers 10^2, 10^3, and ∞. Nusselt number increases with the Rayleigh number as $\mathrm{Nu} \sim \mathrm{Ra}^\gamma$ with γ in the range $0.29 - 0.33$. Péclet number scales as $\mathrm{Pe} \sim \mathrm{Ra}^\zeta$ with ζ in the range $0.57 - 0.61$. The observed results are in general agreement with earlier results. The energy spectrum for Pr = ∞ neither follow the Kolmogorov-Obukhov nor the Bolgiano-Obukhov scaling.

INTRODUCTION

Rayleigh-Bénard convection (RBC) is of tremendous importance in many natural phenomena, e.g., mantle convection, atmospheric circulation, and stellar convection, etc. In RBC, a fluid is placed between two horizontal conducting plates with the lower plate hotter than the upper one. Rayleigh number Ra, a measure of buoyancy force in the system, is defined as Ra = $\alpha g \Delta d^3/\nu\kappa$, where g is gravitational acceleration, Δ and d are the temperature difference and the distance between horizontal plates, respectively, and α, ν, and κ are thermal expansion coefficient, the kinematic viscosity, and the thermal diffusivity of the fluid, respectively. Prandtl number Pr is defined as ν/κ. We solve the Navier-Stokes equation along with the temperature equation, numerically in a box of aspect ratio $2\sqrt{2}$ using pseudospectral code developed by Verma *et al.* [6]. Free-slip boundary condition for the velocity and conducting boundary condition for the temperature field is used for the top and bottom walls. For the vertical walls, however, periodic boundary condition is utilized for both the fields. We conducted numerical experiments for the Prandtl numbers 10^2, 10^3, and ∞ and Rayleigh numbers in the range $10^4 - 10^8$. RK4 method has been utilized for time advancement. Further details of numerical simulations can be obtained in Mishra and Verma [4]. We also performed numerical simulations in a no-slip box for Pr = 10^2 using NEK5000 for comparison.

SCALING OF NUSSELT AND PÉCLET NUMBER

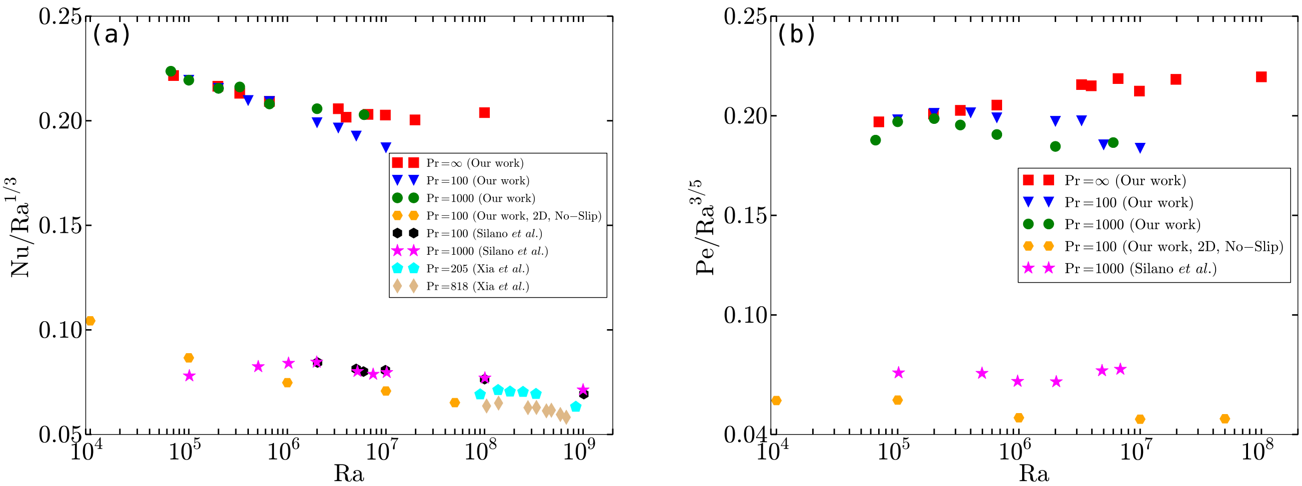

Figure 1. (a)Variation of Normalized Nusselt number (Nu/Ra$^{1/3}$) with the Rayleigh number. Our data appears to scale well with $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$ and also in good agreement with the earlier results [5, 7]. (b) Variation of Normalized Péclet number (Pe/Ra$^{3/5}$) with the Rayleigh number. Our data appears to scale well as $\mathrm{Pe} \sim \mathrm{Ra}^{3/5}$ and in general agreement with earlier result [5].

We compute Nu and Pe as a function of Ra from our simulation data. Figure 1(a) illustrates normalized Nusselt number Nu/Ra$^{1/3}$ as a function of the Rayleigh number. For Pr = ∞, we observe $\mathrm{Nu} = (0.21 \pm 0.03)\mathrm{Ra}^{0.33\pm0.008}$. Furthermore, for Pr = 10^2 and 10^3, we observe $\mathrm{Nu} = (0.34 \pm 0.02)\mathrm{Ra}^{0.29\pm0.003}$ and $\mathrm{Nu} = (0.27 \pm 0.02)\mathrm{Ra}^{0.31\pm0.006}$ respectively. These results are consistent with the earlier results obtained by Grossmann and Lohse [2] ($\mathrm{Nu} \sim 0.17\mathrm{Ra}^{1/3}$), Silano *et al.* [5], and Xia *et al.* [7].

We compute Péclet number Pe from our simulation data. Figure 1(b) depicts the normalized Péclet number Pe/Ra$^{3/5}$ as a function of Rayleigh number. For Pr = ∞, we observe $\mathrm{Pe} = (0.20 \pm 0.02)\mathrm{Ra}^{0.61\pm0.006}$. Moreover, for Pr = 10^2 and 10^3, we observe $\mathrm{Pe} = (0.27 \pm 0.08)\mathrm{Ra}^{0.58\pm0.02}$ and $\mathrm{Pe} = (0.21 \pm 0.04)\mathrm{Ra}^{0.59\pm0.01}$. The scalings are in general agreement with the earlier results [2, 5]. The values of Nu and Pe observed in our simulation is larger compared to the corresponding values from Xia *et al.* [7] and from Silano *et al.* [5] due to the smaller frictional force on free-slip walls compared to no-slip walls.

SCALING OF ENERGY SPECTRUM

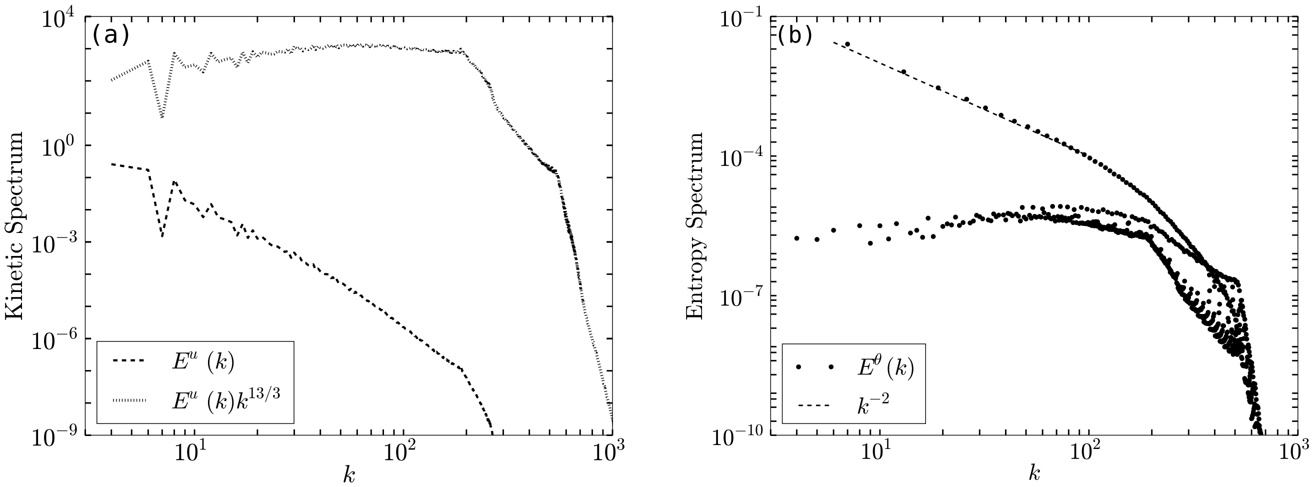

Figure 2. (a) Kinetic spectrum $E^u(k)$ (dashed curve) for Pr = ∞ and Ra = 10^8. Dotted curve is normalized kinetic spectrum $E^u(k)k^{13/3}$. Normalized spectrum appears nearly constant in the inertial range. (b) Entropy spectrum, for Pr = ∞ and Ra = 10^8, exhibits dual branches for smaller wave-numbers. Upper branch of the spectrum represents $\hat{\theta}(0, 0, 2n)$ Fourier modes, which scales well with k^{-2}. The lower branch appears nearly constant in the inertial range.

We compute kinetic spectrum $E^u(k)$ and entropy spectrum $E^\theta(k)$ for Pr = 10^2, 10^3, and ∞. Figure 2(a) shows kinetic spectrum $E^u(k)$ for Pr = ∞ and Ra = 10^8. Kinetic spectrum appears to follow neither Bolgiano-Obukhov [3] nor Kolmogorov-Obukhov [1] scaling for Pr = ∞, but $E^u(k) \sim k^{-13/3}$. Figure 2(b) depicts entropy spectrum $E^\theta(k)$, which contains two branches at smaller wave numbers. Upper branch represent $\hat{\theta}(0, 0, 2n)$ Fourier modes, which follow k^{-2} powerlaw [4]. The lower branch, however, consists all but $\hat{\theta}(0, 0, 2n)$ Fourier modes, which appears to be nearly constant in the inertial range. We observe similar scaling results for Pr = 10^2 and 10^3, both for free-slip and no-slip boundary conditions.

In this abstract we presented the scaling of Péclet and Nusselt number for large and infinite Prandtl numbers. Our scaling results are consistent with the earlier results [2, 5, 7]. We observed that kinetic spectrum scales as $k^{-13/3}$, whereas entropy spectrum is nearly constant in the inertial range.

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