ENERGY SPECTRUM FOR QUASI-STATIC MHD FOR HIGH INTERACTION PARAMETERS

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Abstract We study the kinetic energy spectrum for quasi-static MHD for large interaction parameters. Simulations have been performed in a box for interaction parameters in the range $0-222$. We observe that the kinetic energy spectrum steepens with increasing interaction parameter. We also observe inverse cascade of energy and quasi two-dimensionalization of flow field for large interaction parameters.

Flow of liquid metals under high external magnetic fields can be studied using quasi-static approximation [6, 3, 4]. In this abstract we investigate flows for magnetic Reynolds and Prandtl numbers under the limit $\mathrm{Re}_m \to 0$ and $\mathrm{Pr}_m \to 0$, using direct numerical simulations (DNS).

Governing equations for liquid metal MHD under quasi-static approximation [4] are,

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\frac{1}{\rho}\nabla p - \frac{\sigma {B_0}^2}{\rho}\frac{1}{\nabla^2}\frac{\partial^2 \mathbf{u}}{\partial z^2} + \nu\nabla^2\mathbf{u} + \mathbf{F}^{\mathbf{u}}, & (1)\\ \nabla\cdot\mathbf{u} &= 0, & (2) \end{aligned}$$

where $\mathbf{u}$ is the velocity field, p is the pressure, ρ is the density, ν is the kinematic viscosity, σ is the conductivity, and $\mathbf{F}^{\mathbf{u}}$ is the forcing. The uniform external magnetic field $\mathbf{B_0}$ is in z-direction. Interaction parameter (N) is a useful non-dimensional number to study quasi-static MHD. It is expressed as the ratio of Lorentz force term and the nonlinear term,

$$N = \frac{\sigma {B_0}^2 L}{\rho u'}, \qquad (3)$$

where L is the integral length scale, and u' is rms of fluctuating velocity.

We perform forced simulations of MHD flow in a cubical box of length 2π with a grid resolution of 256^3. Periodic boundary condition is used for all sides to study the bulk flow and to ignore boundary effects. We use pseudo-spectral methods to perform DNS using *Tarang* [9]. Fourth order Runge-Kutta method is used for time stepping and $3/2$ rule is used for dealiasing.

We investigate the behaviour of kinetic energy spectrum for a range of interaction parameters $(N = 0 - 222)$ by varying the constant external magnetic field. We observe that the kinetic energy spectrum steepens with increasing N as shown in figure 1(a). This is due to increased dissipation caused by the Lorentz force. This trend is consistent with the theoretical model proposed by Verma [8]. For $N = 132$ and 222, the kinetic energy spectrum is completely dissipative and follows an exponential behaviour as shown in figure 1(b).

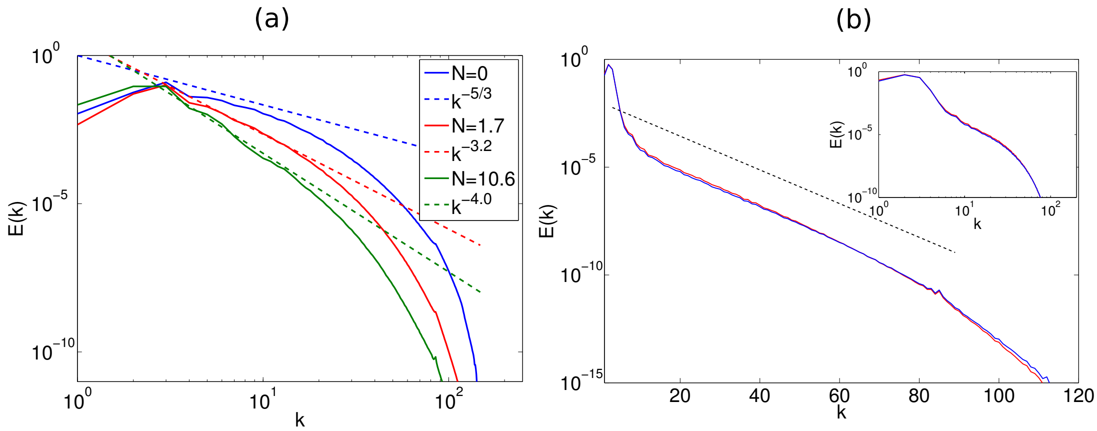

Figure 1. Kinetic energy spectrum for various interaction parameters: (a) Kinetic energy spectrum for $N = 0$, 1.7, and 10.6. The exponent of spectrum increases with increasing interaction parameter; (b) Kinetic energy spectrum for high interaction parameters. For case (b) it is completely dissipative and follows an exponential behaviour ($\exp(-0.18k)$ represented by a dotted black line). Solid blue and pink lines represent $N = 132$ and $N = 222$ respectively.

We study distribution of the kinetic energy and joule dissipation by dividing the spectral space into thin shells and rings [7]. The kinetic energy is concentrated more in the equatorial rings compared to rings located near the pole [1, 7]. We further quantify kinetic energy distribution using Legendre polynomials

$$E(k,\theta) = \sum_{l} a_l P_l(\sin\theta). \tag{4}$$

We compute the coefficients (a_l) using numerical data. We observe that for low N, lower modes (a_l) are dominant while for large N, higher modes (a_l) are dominant. The distribution of the joule dissipation ($\epsilon_J(k,\theta)$) with θ shows that large amount of energy is dissipated in the rings near the equator, and $\epsilon_J(k,\theta)$ vanishes on the equator [5].

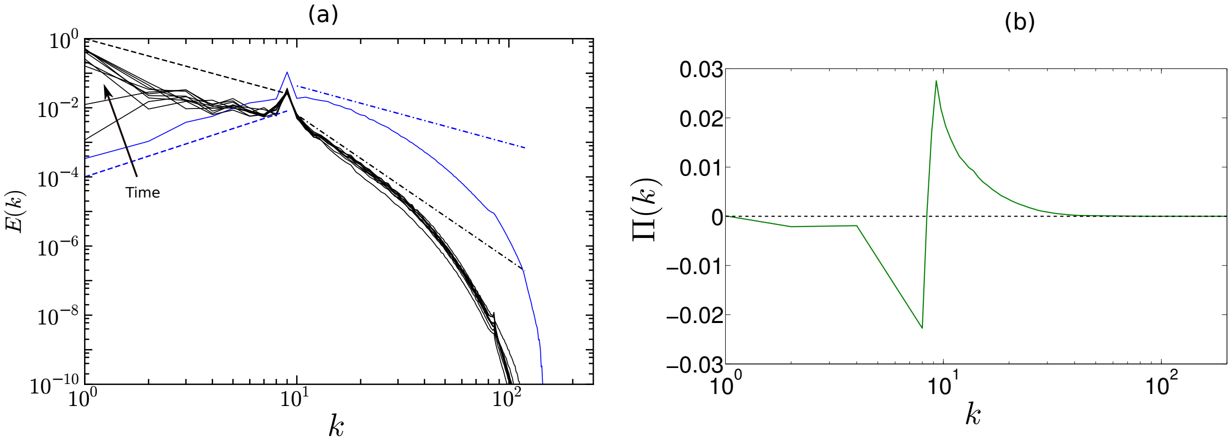

Figure 2. (a) Kinetic energy spectrum at various times for $N = 30$. Solid blue line represents final state for $N = 0$, which is used the initial condition for $N = 30$. Solid black lines represent evolution of kinetic energy spectrum for $N = 30$. $k^{-5/3}$ scaling (black dotted line) in the range $k < k_f$, and $k^{-4.2}$ scaling (black dashed-dotted line) in the range $k > k_f$ are shown. (b) Energy flux averaged over a time after the simulations has reached a statistically steady state is illustrated.

Liquid metal flow is quasi two-dimensional for high N [2, 5, 10]. To understand quasi two-dimensionalization and energy cascade, we performed different set of simulations by forcing wave-numbers in the inertial range ($k_f = 8 - 9$) to explore inverse cascade in wave-numbers below k_f. The time evolution of energy spectrum is shown in figure 2(a). In figure 2(b), we observe negative energy flux for $k < k_f$, which clearly demonstrates an inverse cascade of energy and quasi two-dimensionalization in liquid metal flows at large N.

In this abstract we have studied the steepening of energy spectrum with the increase of interaction parameter. Inverse cascade of energy, negative energy flux, and quasi two-dimensionalization of velocity field are observed at high interaction parameters.

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