NONLINEAR VORTEX STRUCTURES IN BOUNDARY LAYER FLOW

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Abstract We report various nonlinear flow structures for a parallel boundary layer flow, some bifurcate directly from the linearly unstable Blasius flow, others are obtained through an artificial body force $\boldsymbol{F}(y,z)$, of amplitude f_A, using ideas from the self-sustaining process (F. Waleffe. On a self-sustaining process in shear flows. *Phys. Fluids* **9**, 883-900, 1997). The relevant solutions are found by continuing the forced state and smoothly reducing the external force to achieve solutions of the unforced Navier-Stokes equations.

Introduction

Making progress on the understanding of a flow in a boundary layer is of considerable interest since many flow applications or systems appear in a turbulent flow state. A turbulent boundary layer flow imposes a higher resistance on a body compared to the laminar counterpart, hence reducing this hindrance is of great importance for saving energy, e.g. the fuel-consumption of a vehicle. According to the classical linear theory, first undertaken by Tollmien [1] and later by Schlichting [2], the first phase of the turbulence transition, at low free stream turbulence level, is initiated by the exponential amplification of what are now called two-dimensional Tollmien-Schlicting waves at Reynolds number close to 300 (based on the boundary layer scale $\delta=(\nu x/U_\infty)^{½}$, the free-stream speed U_∞ and the kinematic viscosity ν). To describe the turbulent stage the linear theory needs to be abandoned for nonlinear three-dimensional solutions, also called exact coherent structures (ECS). These nonlinear solutions are limit cycles in phase space in the form of large scale flow patterns with statistics similar to those of numerical turbulence [3,4]. This implies that when collected together their statistics can be used as a foundation for reconstructing turbulent flows. Therefore, finding a large set of these solutions is needed to provide us with the essential elements for understanding the mechanics of chaotic flows and to set the basis for a dynamical-system-theory of turbulence. Earlier studies have shown the importance of the exact coherent structures [5-7], and lead to a large effort on finding them in various flow configurations [8-13]. The fact that the exact coherent structures may describe parts of a chaotic flow has implications for future investigations on flow control for managing turbulence.

Definitions

The flow over a flat plate in an isothermal incompressible boundary layer at zero angle of incidence is studied. The plate is considered to be infinite in the streamwise direction x and the spanwise direction z. The leading edge is situated at x=0, and y denote the vertical coordinate. The unit vectors are $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ and the velocity vector is defined as $\boldsymbol{v}=u\boldsymbol{i}+v\boldsymbol{j}+w\boldsymbol{k}$ represented by the streamwise, wall-normal and spanwise component respectively, the pressure as p and time t. The variables are non-dimensionalised using the uni-directional free-stream speed U_∞, density ρ and the length scale $\delta=(\nu x/U_\infty)^{½}$. We search for solutions in the form of finite amplitude perturbations in a parallel Blasius boundary layer flow of the Navier-Stokes equations, they move at a phase velocity of $c\boldsymbol{i}$ and their periodicity in x and z is given by the streamwise wavenumber α and the spanwise wavenumber β. Their numerical expression is:

$$\boldsymbol{u}(x,y,z,t)=\sum_{b=-NX}^{NX}\sum_{j=-NZ}^{NZ}\boldsymbol{u}^{(bj)}(\gamma(y))e^{Ij\beta z}e^{Ib\alpha(x-ct)}=\sum_{b=-NX}^{NX}\sum_{j=-NZ}^{NZ}\sum_{i=0}^{NY}\boldsymbol{u}_{bji}T_i(\gamma(y))e^{Ij\beta z}e^{Ib\alpha(x-ct)}.$$

The T_i is the classical Chebyshev polynomial, $I=(-1)^{½}$ and the γ is the mapping of the truncated physical domain $0 \leq y \leq y_{max}$ to $-1 \leq \gamma \leq 1$. The solutions discovered bifurcate either from an artificial flow state or from the unstable laminar Blasius flow $U(y)\boldsymbol{i} = f_\eta \boldsymbol{i} = (df(\eta)/d\eta)\boldsymbol{i}$. The function $f(\eta)$ is governed by the well-known Blasius equation $f_{\eta\eta\eta} + ½ f f_{\eta\eta}=0$, where the coordinate η is the similarity variable having the formula $\eta=y/\delta$. The total flow is $\boldsymbol{v}=kU\boldsymbol{i} + \boldsymbol{u}$; the governing equations for the perturbation field are forced by a body force $\boldsymbol{F}(y,z)$ of amplitude f_A and are defined as

$$\boldsymbol{u_t}+(kU.\nabla)\boldsymbol{u}+(\boldsymbol{u}.\nabla)kU+(\boldsymbol{u}.\nabla)\boldsymbol{u}+\nabla p-\frac{1}{R}\nabla^2\boldsymbol{u}=f_A\boldsymbol{F}(y,z).$$

For solutions bifurcating from the Blasius flow we have f_A≡0. R is the Reynolds number defined as $R=U_\infty\delta/\nu$. We have no-slip boundary conditions for $\boldsymbol{u}$ at the plate and asymptotic conditions at $y=y_{max}$ or $\boldsymbol{u}_y^{(bj)}(y) + (b^2\alpha^2+j^2\beta^2)^{½}\,\boldsymbol{u}^{(bj)}(y)=0$, where $\boldsymbol{u}^{(bj)}(y)$ is the function seen in the numerical expression above. In the free-stream we have uniform flow or $\boldsymbol{v}=(1,0,0)$. The coefficient k is an amplitude which serves to maintain the uniform flow at $y=y_{max}$ constant in the presence of finite amplitude ECS [14]. Performing a linear stability analysis of U and mapping out the linear solutions in α-R space one gets the neutral curves shown in figure 1. To find solutions bifurcating from an artificial flow

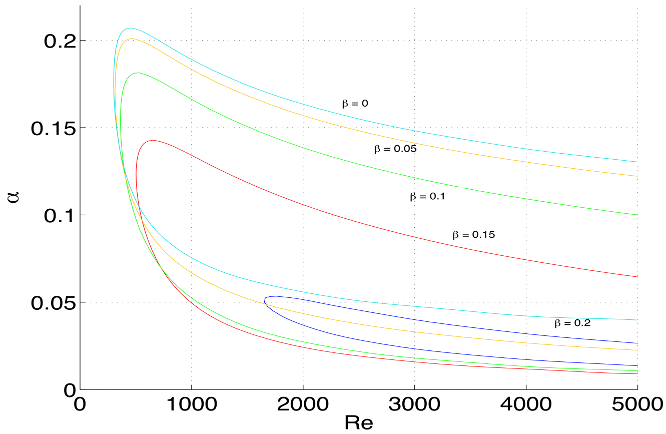

Figure 1. Neutral curves of selected perturbations of spanwise wavenumbers β=0-0.20. Increasing β lowers (in α), narrows and displaces (to larger Reynolds numbers) the envelope of the linearly unstable solutions, making it difficult to find solutions for β>0.20.

state we use the idea behind the self-sustaining process (SSP) [15] and force solutions by $\boldsymbol{F(y,z)}$. To find the relevant flow states the amplitude f_A is gradually brought to zero yielding e.g. solutions such as those displayed in figure 2.

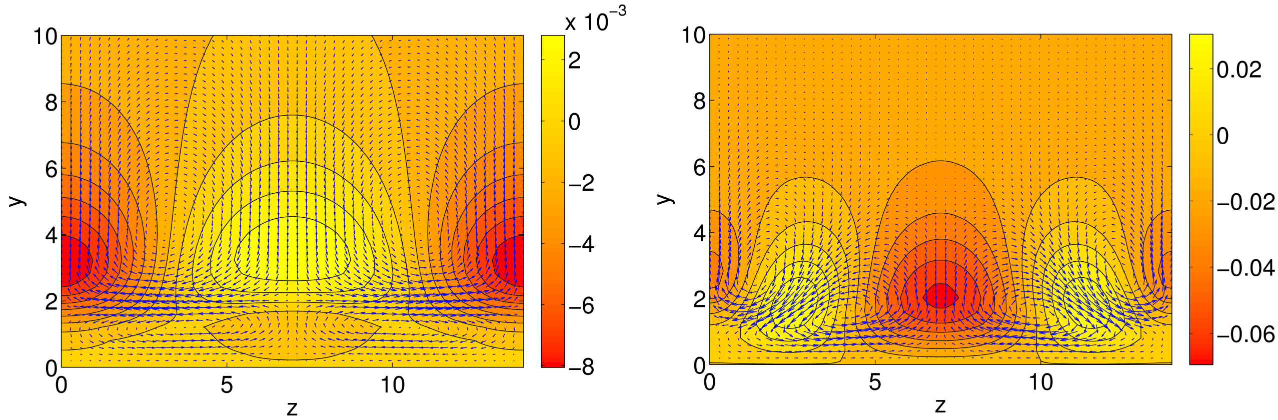

Figure 2. Mean velocity fields of the nonlinear ECS at Re=332 (low amplitude) and Re=390 (larger amplitude) for the streamwise and the spanwise wavenumber α=0.20 and β=0.45. The Blasius boundary layer thickness is situated at y=5.

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