DNS OF INHOMOGENEOUS TURBULENCE UNDER ROTATION

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Abstract Rotating turbulent flow is routinely encountered in geophysical environment, such as in cyclones and tornadoes. The number of investigations on inhomogeneous rotating turbulence is relatively sparse as compared to its homogeneous counterpart although they are more common in nature. The present study involves Direct Numerical Simulation (DNS) of a 'cloud' of turbulence with quiescent fluid on either side, under background rotation. The motivation behind the study is to understand the role of inertial waves in inhomogeneous rotating turbulence. In the results obtained, helicity - a quantity representative of the degree of entanglement of the vortices, is found to propagate in a way typical of inertial waves. The thickness of the growing cloud is found to scale linearly with time.

A 'CLOUD' OF TURBULENCE UNDER ROTATION

A 'cloud' of turbulence is defined, for the purposes of our study, as a confined region of turbulence in a quiescent environment, in which inhomogeneity is present along one axis. It mimics the flow environment in the stratosphere where inertial waves have been observed under Earth's rotation [1], although effects like cloud condensation are not considered. This is an unbounded initial value problem in the absence of any physical boundaries. The Coriolis force is assumed to be the only body force present so that the effect of rotation can be investigated in isolation. A similar cloud under rotation was investigated by Staplehurst [2] using flow visualization with reflective particles (Pearlescence) in a rotating tank at a Rossby number* (Ro) ~ 1. Columnar flow structures were observed to emerge and fill the tank over time and their linear growth was attributed to inertial waves. These results are supported by the observations of Kolvin *et al*. [3] who used Particle Image Velocimetry (PIV) in a rotating tank to study the propagation of inertial waves. Kolvin *et al.* concluded that the inertial waves dictate the flow dynamics only in a linear time scale, after which the non-linear dynamics takes over. However, the exact role of inertial waves in the formation of columnar structures remains unclear.

Compared to experiments, numerical simulations provide greater control over the initial conditions. In the present study, the first of its kind for rotating turbulence, an initial condition similar to those in the experiments [2, 3] is simulated. As the first step, a 256^3 resolution pseudo-spectral DNS of isotropic turbulence in a periodic cube (length 2π) is conducted starting with a velocity field obtained from input energy spectrum; and run till the turbulence is fully developed. Subsequently, a spatial filter is used to confine the turbulence to the middle of the periodic cube so that it is free to evolve in the vertical z direction. This process is illustrated in Figure 1.

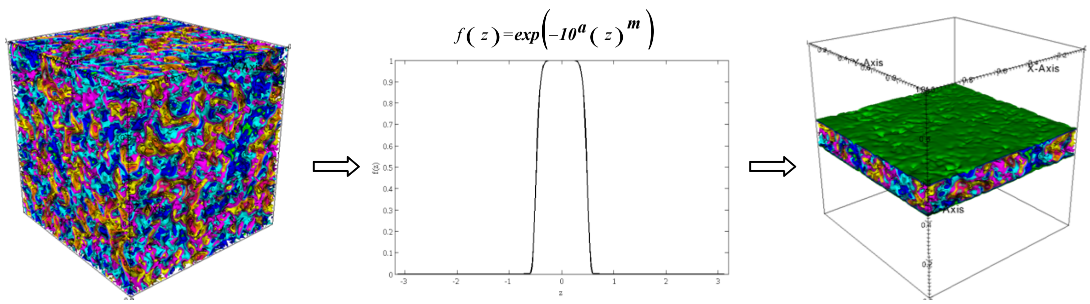


Figure 1. Confining fully developed isotropic turbulence to the center of the periodic cube using a spatial filter

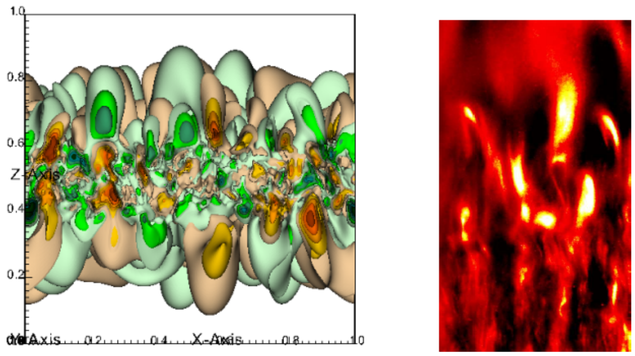
The DNS code used in the present study is based on the standard Rogallo's algorithm and uses an integrating factor to account for the rotation term [4]. This code is first validated using the analytical solution available for a Gaussian eddy under rotation [5]. Starting with the confined velocity field (Figure 1), a 256^3 resolution DNS is conducted with rotation about the vertical z axis at Ro=0.1 (l=0.31; Re_l=130; L_{BOX}/l = 20.3).

DNS RESULTS

In the DNS results, vertical flow structures are observed to emerge from the 'cloud' of turbulence as shown in the velocity (u_z) iso-surfaces (Figure 2a). It is found that the time it takes for these structures to reach the cube boundary is

* Rossby number is $Ro = u/2\Omega l$, where u and l are the characteristic velocity and length scales, respectively; Ω is the rotation rate.

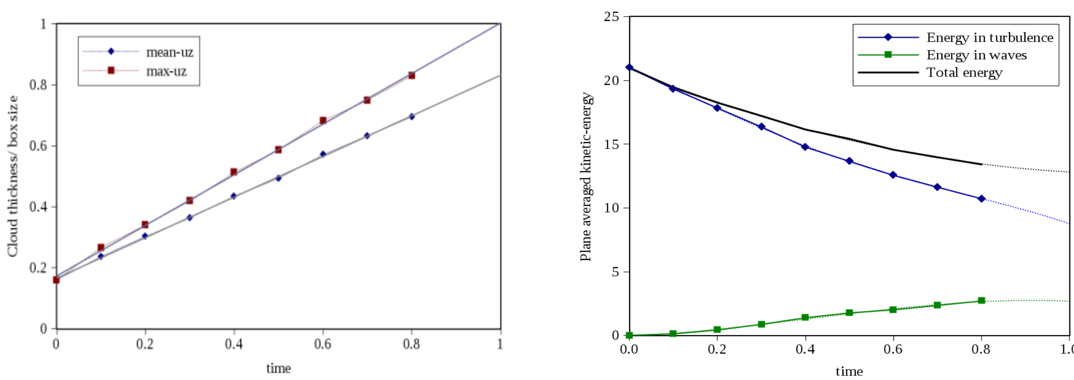
very close to the saturation time estimated from the group velocity of inertial waves, $t_s \sim \pi/C_g$ where $C_g \sim 2\Omega l$ [5]. A qualitative comparison of the u_z iso-surfaces obtained from DNS with Pearlescence visualization intensity image [2] is shown in Figure 2 with good agreement (it may be noted that in the case of experiments the flow can evolve only along $+z$ where as it can evolve in both $+/-z$ in the case of DNS). An interesting result is described in terms of the propagation of helicity density defined as $h=\boldsymbol{u.\omega}$. It is observed that the negative h propagates in the positive z direction whereas the positive h propagates in the negative z. This characteristic is a property of inertial waves [6]. Thus, it indicates that inertial waves are the cause of formation of columnar structures in the case of inhomogeneous rotating turbulence.



(a) (b)

Figure 2: Qualitative comparison of DNS (front-view of u_z iso-surface) with Pearlescence visualization [2]

The mean and maximum thicknesses of the growing cloud are estimated using boundary tracking method applied to the u_z iso-surfaces. A linear increase in thickness with time is observed (Figure 3a) indicating that all the flow structures grow linearly with time. Using particle tracking, the kinetic energy transferred to the inertial waves is estimated to be about 13% of the initial total kinetic energy in the turbulence cloud, where as the energy dissipated is about 36% , between t=0 and 0.8 (when the waves reach the cube boundary), as shown in Figure 3b.



(a) (b)

Figure 3: (a) Linear growth of cloud thickness (b) Energy transfer to inertial waves

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