TURBULENT BURSTS AND TORQUE MAXIMA IN TAYLOR-COUETTE FLOW

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Abstract In Taylor-Couette flow with sufficiently strong counter-rotating cylinders, the turbulence near the outer cylinder shows intermittent oscillations in form of turbulent bursts. We present this phenomenon in direct numerical simulations for radius ratios $\eta = 0.5$ and 0.71 and identify a critical value in the rotation ratio μ for its onset. We propose a physical model for this critical value that also rationalizes the observed agreement with the rotation ratio of the torque maximum. While this model conforms well with the observations for these radius ratios, it most likely has to be refined or replaced for $\eta \to 1$, as further simulations for $\eta \geq 0.9$ show.

NUMERICAL SIMULATIONS

The flow between two independently rotating cylinders (Taylor-Couette flow) becomes turbulent either after a sequence of instabilities or via a sub-critical transition scenario [1]. In the latter case, the turbulence is not always space-filling but can form turbulent spots and spirals for counter-rotating cylinders. In this regime, Coughlin & Marcus [2] described a flow state that is additionally intermittent in the radial direction in form of turbulent bursts which were also observed at much higher Reynolds numbers of $\sim 10^6$ in recent experiments [3].

To investigate these radial turbulent fluctuations, we present direct numerical simulations of Taylor-Couette flow using a spectral code [4] for domains that are periodic in the axial direction. For a constant differential rotation between the cylinders, characterized by the shear Reynolds number [5]

$$Re_S = \frac{2\, r_o\, r_i (r_o - r_i)}{r_o + r_i}\, \frac{(\omega_i - \omega_o)}{\nu} = 2.0 \cdot 10^4 \ ,$$

we study the influence of the mean rotation on the flow behavior by realizing various rotation states defined by the rotation ratio $\mu = \omega_o/\omega_i$. Here, r_i and r_o are the radii and ω_i and ω_o the angular velocities of the inner and outer cylinder, respectively, and ν denotes the kinematic viscosity of the fluid. For sufficiently strong counter-rotation, with $\mu < \mu_c < 0$, we find a separation of the flow into an outer region with intermittent oscillations of the turbulent intensity, i.e. the aforementioned bursting behavior, and an inner region that is permanently turbulent. For less counter-rotation, $\mu_c < \mu < 0$, this radial separation vanishes. Based on our numerical simulations for $\eta = 0.5$ and 0.71, we identify the η-dependent critical rotation ratio μ_c for the onset of enhanced outer fluctuations (turbulent bursts). As also found experimentally [3], the onset of the bursting activity at moderate counter-rotation coincides with the rotation ratio at which a maximum in the torque is observed [6, 7, 8]. We complement these recent torque studies by a reanalysis of Wendt's torque measurements [9] that also reveals a maximum for moderate counter-rotation as shown in Figure 1 and in [10].

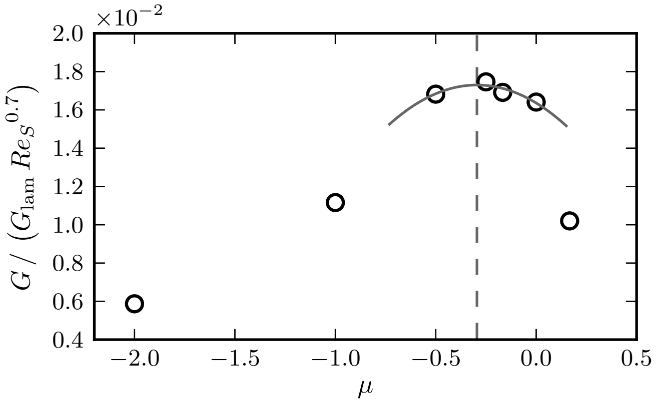

Figure 1. Reanalysis of torques G measured by Wendt for $\eta = 0.680$ [9]: We here employ the empirical observation [5, 6, 7] that the torque is well described by the factorization $G/G_{\mathrm{lam}} = f(\mu)\, g(Re_S)$ with the laminar torque G_{lam}. As Wendt found an effective scaling $g(Re_S) \sim {Re_S}^{0.7}$, the additional compensation by ${Re_S}^{0.7}$ results in the rotation-dependence $f(\mu)$ of the torque which has been averaged in the range $7.8 \cdot 10^4 < Re_S < 1.3 \cdot 10^5$ for each rotation ratio independently (open circles). The solid line indicates a quadratic fit to the largest data points and gives rise to a torque maximum at $\mu_{\max} = -0.295 \pm 0.113$ marked by a dashed line.

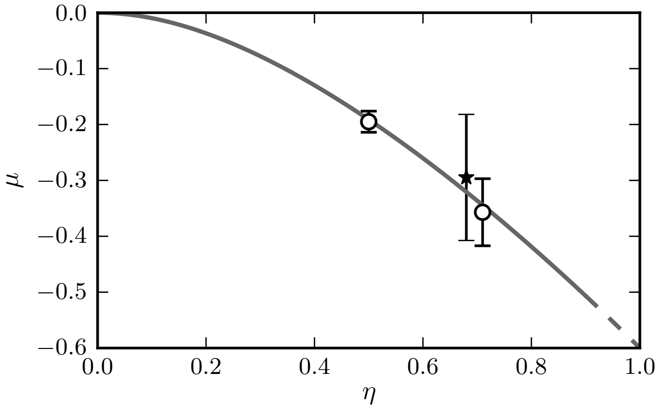

Figure 2. Rotation ratio of torque maxima in dependence on the radius ratio: The line indicates the estimate $\mu_{\mathrm{pred}}(\eta)$ obtained from the physical model for the onset of enhanced fluctuations [10]. It agrees well with the torque maxima from direct numerical simulations (open circles) and Wendt's torque maximum extracted in Figure 1 (star). Since the basic concept of the physical model is not valid for $\eta \to 1$, we indicate the resulting uncertainty of the predictive estimate $\mu_{\mathrm{pred}}(\eta)$ by a dashed line for $\eta \geq 0.9$.

PHYSICAL MODEL FOR THE ONSET OF ENHANCED FLUCTUATIONS

To explain the dependence of the fluctuation onset μ_c on the radius ratio η and its connection to the occurrence of a torque maximum, we propose a physical model that is based on the separation of the flow into an actively driven inner region and a stable outer region that is susceptible to turbulent bursts. The inviscid stability calculation implies that a separation into an unstable inner region that reaches up to the radius of vanishing angular velocity r_n and a stable outer region for $r_n < r < r_o$ occurs for $\omega_i > 0$ and $\omega_o < 0$. This suggests that the enhanced fluctuations set in for any $\mu < 0$ independent of η which is in contrast to our empirical observations. However, considering the fact that the neutral surface does not define a hard boundary condition, the turbulent fluctuations from the interior can protrude into the stable region. The distance they can cover is a multiple $a(\eta)$ of the inviscidly unstable region, and can be estimated by the factor $a(\eta) \in [1.4, 1.6]$ calculated by Esser & Grossmann [11] within their linear stability calculations. Using this value we can predict the critical rotation ratio $\mu_{\mathrm{pred}}(\eta)$ where the separation of the flow occurs and where, thus, enhanced outer fluctuations have to set in [10]. To connect this to the observed maximum in the torque, we demonstrate that with the onset of counter-rotation the strength of the Taylor vortex flow contribution to the turbulent flow field and also the overall drag increases. This increase ends when the intermittency in the outer boundary layer sets in. Accordingly, we can use $\mu_{\mathrm{pred}}(\eta)$ also to predict the location of the torque maximum. The results are in excellent agreement with the location of the maxima identified in the currently available numerical simulations and experimental observations, including the torque maximum revealed in the reanalyzed data of Wendt, see Figure 2.

Since our predictive model relies on the separation of the flow into an unstable inner and intermittent outer region, it has to be refined or replaced in the limit $\eta \to 1$ where this separation vanishes. To study this limit, we currently perform numerical simulations for $\eta \geq 0.9$ and compare them to the limiting situation of rotating plane Couette flow for $\eta = 1$.

References

[1] C. D. Andereck, S. S. Liu, and H. L. Swinney. Flow regimes in a circular Couette system with independently rotating cylinders. *J. Fluid Mech.* **164**: 155–183, 1986.

[2] K. Coughlin and P. S. Marcus. Turbulent bursts in Couette-Taylor flow. *Phys. Rev. Lett.*, **77** (11): 2214–2217, 1996.

[3] D. P. M. van Gils, S. G. Huisman, S. Grossmann, C. Sun, and D. Lohse. Optimal Taylor-Couette turbulence. *J. Fluid Mech.* **706**: 118–149, 2012.

[4] A. Meseguer, M. Avila, F. Mellibovsky, and F. Marques. Solenoidal spectral formulations for the computation of secondary flows in cylindrical and annular geometries. *Eur. Phys. J. Special Topics* **146**: 249–259, 2007.

[5] B. Dubrulle, O. Dauchot, F. Daviaud, P.-Y. Longaretti, D. Richard, and J.-P. Zahn. Stability and turbulent transport in Taylor-Couette flow from analysis of experimental data. *Phys. Fluids* **17** (9): 095103, 2005.

[6] M. S. Paoletti and D. P. Lathrop. Angular momentum transport in turbulent flow between independently rotating cylinders. *Phys. Rev. Lett.* **106**: 024501, 2011.

[7] D. P. M. van Gils, S. G. Huisman, G.-W. Bruggert, C. Sun, and D. Lohse. Torque scaling in turbulent Taylor-Couette flow with co- and counter-rotating cylinders. *Phys. Rev. Lett.* **106**: 024502, 2011.

[8] H. J. Brauckmann and B. Eckhardt. Direct numerical simulations of local and global torque in Taylor-Couette flow up to Re=30000. *J. Fluid Mech.* (accepted), arXiv:1206.1286, 2012.

[9] F. Wendt. Turbulente Strömungen zwischen zwei rotierenden konaxialen Zylindern. *Ingenieur-Archiv* **4**: 577–595, 1933.

[10] H. J. Brauckmann and B. Eckhardt. Intermittent boundary layers and torque maxima in Taylor-Couette flow. *Phys. Rev. E* (submitted), arXiv:1210.6075, 2012.

[11] A. Esser and S. Grossmann. Analytic expression for Taylor-Couette stability boundary. *Phys. Fluids* **8** (7): 1814–1819, 1996.