# TURBULENT-LAMINAR BANDS IN PLANE POISEUILLE FLOW

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*Abstract* Turbulent-laminar banded patterns near transition in plane Poiseuille flow are studied numerically.

Transition to turbulence in wall-bounded shear flows is one of the most fundamental unexplained phenomena in fluid dynamics. Near transition, flows exhibit regular patterns of turbulent-laminar stripes whose wavelength is much larger than the gap between the bounding plates and which are oriented at an angle $\theta$ with respect to the streamwise direction. These have been studied extensively in counter-rotating circular Couette flow [1, 2, 3] and in plane Couette flow [3, 4, 5]. Turbulent-laminar banded patterns also exist in plane Poiseuille flow [6]. We simulate these by integrating the 3D Navier-Stokes equations using the parallelized Fourier/spectral-element code Prism [7]. In order to more easily follow flow features and patterns, we use a translating reference frame: the bounding plates are assigned a (negative) velocity and the streamwise flux is set to zero at each timestep by choosing an appropriate streamwise pressure gradient via a Green's function technique. With these conventions, the boundary conditions, enforced streamwise flux, and laminar velocity profile are:

$$u(y=\pm 1) = -2/3 \qquad \int_{-1}^{1} dy\; u(y) = 0 \qquad u_{\text{laminar}}(y) = -2/3 + (1-y^2) \tag{1}$$

We use the same *tilted geometry* for this simulation of plane Poiseuille flow as was used in previous studies of plane Couette flow [4]. The periodic direction $L_z$ is taken to be large and aligned with the expected pattern wavevector while the periodic direction $L_x$ is of the minimal size necessary to sustain turbulence. We take $\theta = 24°$ and use $N_x \times N_y \times N_z = 81 \times 41 \times 512$ gridpoints or basis functions to represent a domain of size $L_x \times L_y \times L_z = 10 \times 2 \times 40$.

Figure 1 describes a simulation in which the Reynolds number is decreased from Re=2000. Figure 1a shows streamwise velocity profiles $u(y)$ that are typical of turbulent or laminar flow [8], spaced at time intervals of 2000 in advective units. Figure 1b shows timeseries of the spanwise velocity $w(t)$ along a line in the midplane $x = y = 0$, spaced at $z$ intervals of $L_z/32$. A banded pattern exists over the range $1550 \geq Re \geq 850$. The pattern travels more slowly than the mean flux when $Re > 1400$, i.e. it moves leftwards in the zero-mean-flux frame defined by (1), and more quickly when $Re < 1400$. For $Re \geq 1100$, two bands are present so that the wavelength is 20, whereas one band is present for $Re \leq 1050$. Figures 1c,d show that these features can be extracted quantitatively via temporal averages of the modulus $|\hat{w}_m(t)|$ and phase $z_m(t)$ of the $z$-Fourier transform:

$$w(z,t) = \sum_m \hat{w}_m(t) e^{imz2\pi/L_z} = \sum_m |\hat{w}_m(t)| e^{im(z-z_m(t))2\pi/L_z} \tag{2}$$

Figure 2 shows the mean flow (averaged over $L_x$ and 2000 time units) of a pattern at $Re = 1400$. Figure 2a shows the chevron appearance of the mean spanwise velocity. The pattern of signs in the mean cross-channel velocity in figure 2b indicates the presence of two layers of cells, superposed over the gap. Indeed, it has been proposed [9] that plane Poiseuille flow can be viewed as two superposed plane Couette flows. Scaling by the quarter-gap would make the pattern wavelength of 20 at higher Reynolds numbers consistent with the wavelength of 40 (scaled by the half-gap) observed for plane and circular Couette flow. An appealing feature of plane Poiseuille flow is that it is intermediate between plane Couette flow, which has two extended horizontal directions, and of pipe flow, in which the shear changes sign and in which the understanding of transition has recently greatly advanced [10]. More detailed analysis of this flow is underway.

## References

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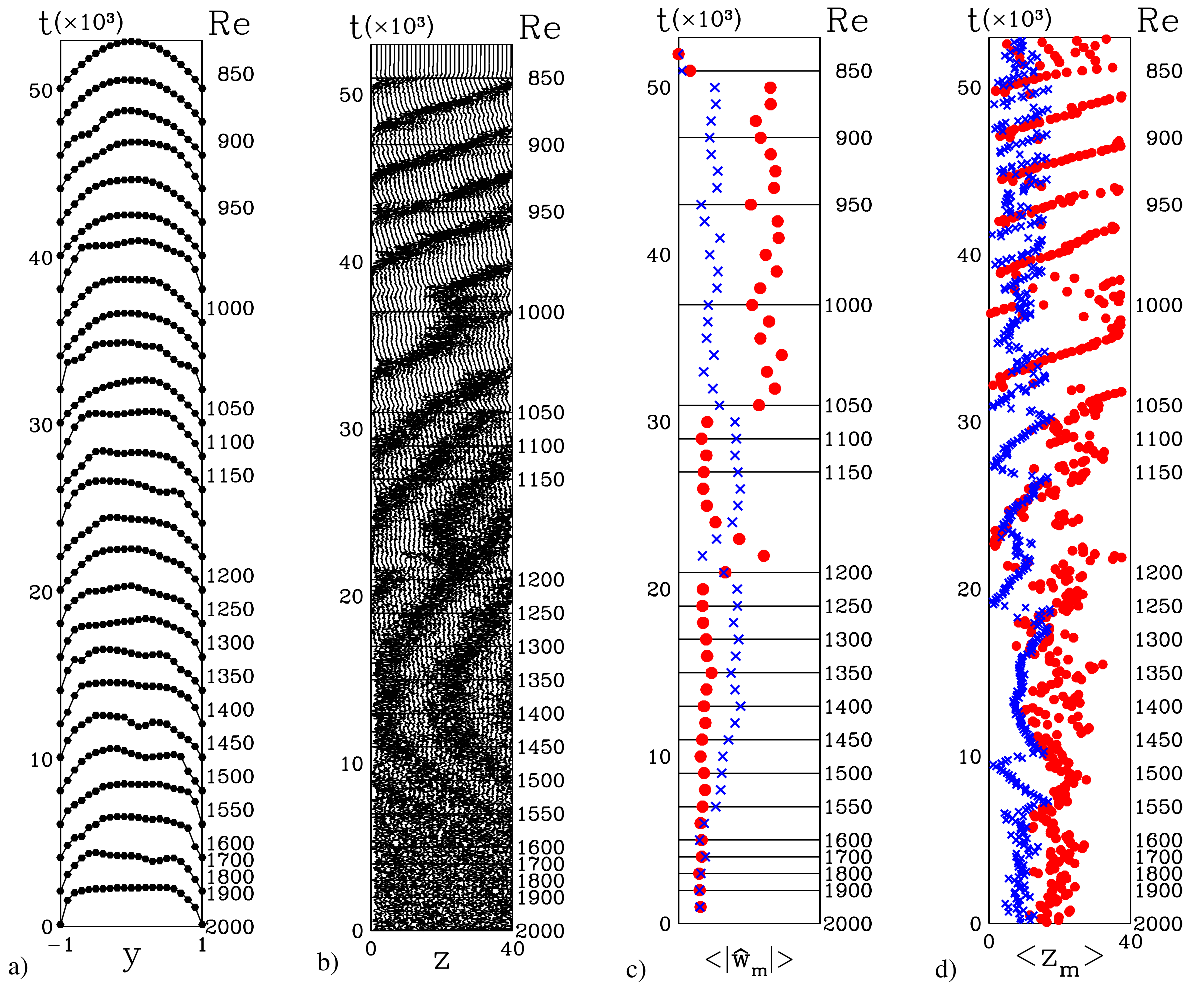


**Figure 1.** Simulation of plane Poiseuille flow in a domain of size $L_x \times L_y \times L_z = 10 \times 2 \times 40$ in which the $z$ direction is aligned along the pattern wavevector. The Reynolds number is lowered from 2000 to 850 in discrete steps as indicated along right edge.
a) Instantaneous streamwise velocity profiles $u(y)$ along line $x = z = 0$ at times separated by 2000.
b) Instantaneous spanwise velocity $w(t)$ along line in midplane $x = y = 0$ at $z$ locations separated by $L_z/32$.
c) Time-averaged modulus of $z$-Fourier transform of spanwise velocity $\langle|\hat{w}_m(t)|\rangle \equiv \frac{1}{1000}\int_{t'=0}^{1000} dt' \; |\hat{w}_m(t+t')|$.
d) Time-averaged phase of $z$-Fourier transform of spanwise velocity $\langle z_m(t)\rangle \equiv \frac{1}{100}\int_{t'=0}^{100} dt' \; z_m(t+t')$.
For c,d), blue crosses represent wavenumber $m = 2$ with wavelength 40; red disks represent wavenumber $m = 1$ with wavelength 20.

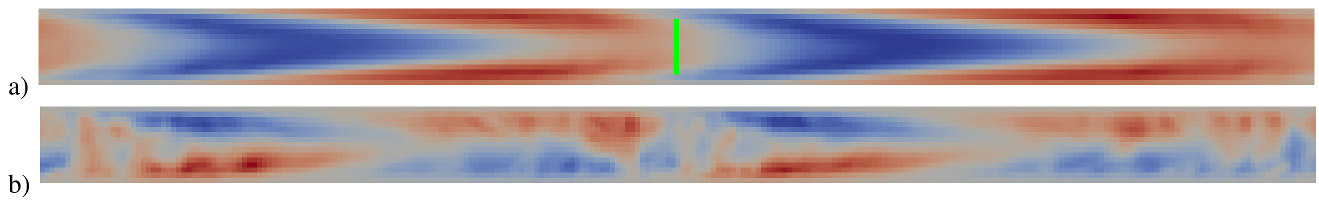


**Figure 2.** Mean flow $\left(\int_{t=13\,000}^{15\,000} dt \int_{x=0}^{L_x} dx\right)$ of turbulent-laminar banded state in the $(z, y)$ plane at Reynolds number 1400.
a) Spanwise velocity with scale [-0.05,0.03]. b) Cross-channel velocity with scale [-0.01,0.01].