FARADAY SURFACE RIPPLE FORCED 2D TURBULENCE

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Abstract
We report new experimental results which support that the particle motion on the surface of Faraday waves share similarities with the fluid motion in 2D turbulence [1]. These results extends the original findings reported in [2]. Since it is clear that the particle motion is not strictly 2D and some of the 2D turbulence theory′s assumptions are violated in Faraday waves, we investigate how robust this analogy is by exploring in depth several features of 2D turbulence such as: the $-5/3$ kolmogorov scaling for the energy spectrum of the velocity fluctuations, the spectral localness of the energy injection, the 3rd order velocity structure function.

CINEMATICS OF THE FARADAY WAVES AND OF THE HORIZONTAL PARTICLES TRANSPORT

Faraday waves are generated in a vertically shaken container (circular, diameter=178mm), and in the deep water conditions. The forcing is monochromatic and varied in the range of frequencies $f_0 = 30 - 110$ Hz which results in Faraday wavelengths λ in the range $\lambda = 5 - 12.5$ mm. The dominant frequency of the excited surface ripples is at the first subharmonic of the excitation frequency, $f = f_0/2$. A diffusing light imaging technique is used to visualize the surface ripples Fig. 1 along with the horizontal motion of floating tracer particles Fig. 1(a-c).
It has recently been shown that the Faraday ripples properties are consistent with those of an ensemble of oscillating solitons, or oscillons, rather than waves [3]. Figure 1(c) shows examples of a few oscillons' trajectories to be compared with the particle tracks measured using the same tracking code. The horizontal oscillons motion is random in time but confined to a disordered lattice as seen from the black trajectories shown in Fig. 1(c) [4]. The particles wander erratically as well, but their trajectories show substantially larger excursions in comparison to oscillons. A particle usually visits several oscillon sites which randomize its trajectory [5]. In the case of a perfect oscillonic crystal, in which nodes relative position is frozen, the horizontal mobility of the particles is cancelled out, confirming that the particle transport is strongly related to oscillon mobility. Such crystals are obtained by adding minute amounts of bovine serum to water [3].

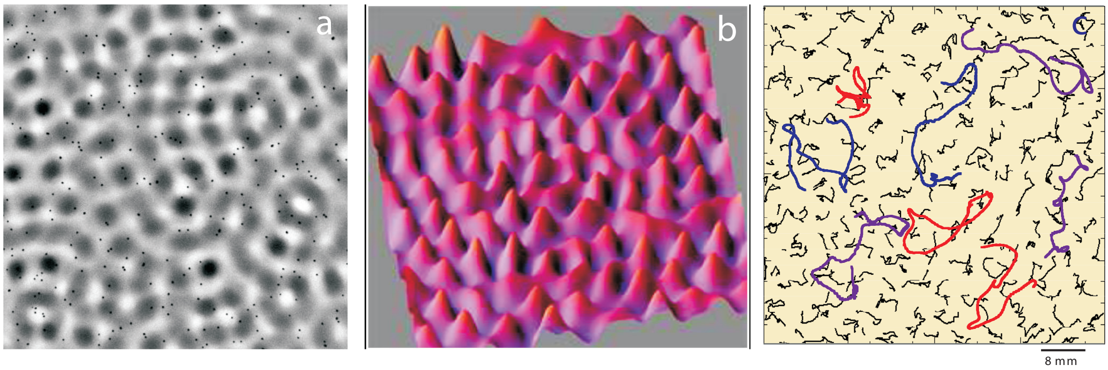

Figure 1. (a) Diffusive light image of the surface elevation with floating particles on the surface at a vertical acceleration a =1.2 g and $f_0 = 60Hz$. The solution is a mixture of distilled water and 2% skim milk. The peaks and troughs of the oscillons appear as dark and white blobs. (b) Measured surface elevation snapshot. (c) Trajectories of oscillons having a phase maximum in (a) (black trajectories) and trajectories of 8 particles (colored) filmed for 2 s (200 periods of the shaker oscillations)

TURBULENT FEATURES OF THE HORIZONTAL TRANSPORT OF PARTICLES

. spectral localness of the forcing:
Though the oscillon lattice in Fig. 1(b) seems disordered, the wave number spectrum of the ripples elevation is very narrow, as seen in Fig. 2(a). This is consistent with the fact that oscillon motion is restricted in space to about half of the lattice characteristic length [4]. To generate 2D turbulence one needs to inject energy into the horizontal fluid motion at some intermediate range of scales in a localized wave number domain. The horizontal mobility of oscillons is responsible for the particle motion on the water surface, and this forcing mechanism is restricted to a narrow $k-$domain.

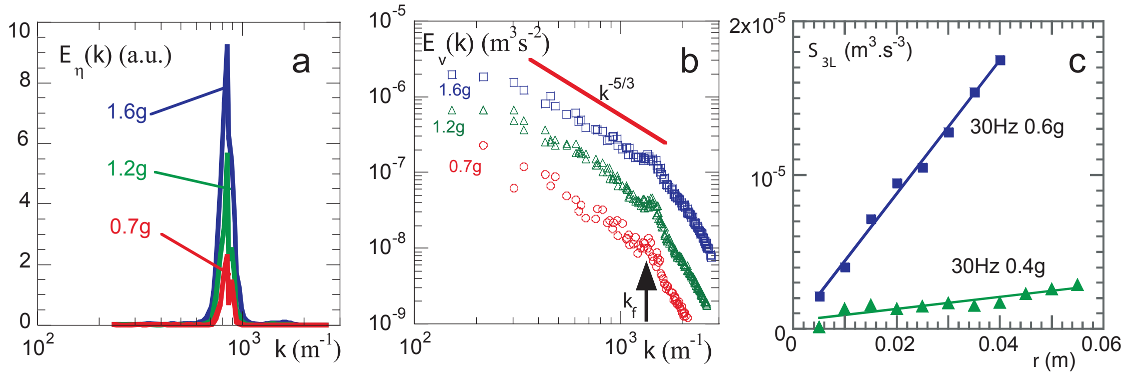

Figure 2. Wave number spectra of the surface elevation (a), and of the kinetic energy of the horizontal flow (b) at different levels of the vertical acceleration ($a = 0.7$ g, 1.2 g, and 1.6 g) and $f_0 = 60Hz$. (c) Third moment of the longitudinal velocity increments in the surface ripple driven turbulence at $f_0 = 30$ Hz and at (a=0.4g,0.6g).

. Kolmogorov-Kraichnan energy spectrum

Particle Image Velocimetry (PIV) is used to characterize the spatial structure of the velocity fluctuations of the horizontal motion of the particles. Figure 2(b) shows energy spectra of the horizontal velocity fluctuations measured at $f_0 = 60$ Hz at three vertical acceleration levels, $a = 0.7$g, 1.2g, and 1.6g (the threshold of Faraday waves generation is $a = 0.6$g at 60 Hz). The spectra scaling is close to the Kolmogorov-Kraichnan theory power law of $E_\nu(k) \propto k^{-5/3}$ at wave numbers $k \leq 1500$ m^{-1}. At higher wave numbers, $k > 1500$ m^{-1}, spectra are typically steeper than expected from the direct enstrophy cascade fit of k^{-3}, probably due to effects of three dimensionality at small scales [2]. Similar spectra were obtained at several frequencies of the vertical vibrations: 30 Hz, 45 Hz, 60 Hz. [1]

If we define the turbulence forcing wave number from the position of the kink on the turbulence spectrum, e.g. 1550 m^{-1} at 60 Hz, Fig. 2(b), then it appears that the turbulence forcing wave number is approximately twice larger than the surface ripple wave number, as seen from the surface ripple spectrum, Fig. 2(a). The energy injection scale for the horizontal particle transport is thus given by the oscillon size rather than by a period of the Faraday wave [3].

. 3^{rd} order velocity structure function

In the inertial range of turbulence, there is a relation between the third order moment $S_{3L} =< [v(l+r) - v(l)]^3 >$ of the velocity increments across a distance r in a flow (the angular brackets denote statistical and time averaging over all possible positions l within the flow field) and the spectral energy flux ε as Kolmogorov first noticed via a dimensional analysis. This relation in 2D turbulence reads as: $S_{3L} = \frac{3}{2}\varepsilon r$. Fig. 2(c) shows third order structure functions computed from the velocity fields measured in the surface ripple driven turbulence ($f_0 = 30$Hz) at two different vertical accelerations. In both cases S_{3L} is a positive linear function of the separation distance r. The values of the spectral energy flux ε are in agreement with the spectral power densities derived from the associated spectra.

These facts supports the existence of an inverse energy cascade fueled by the Faraday 'oscillons'. Interestingly the inverse energy cascade, initially thought of as a process intrinsic to idealized 2D turbulence seems to exist in a variety of physical systems where its presence could not possibly be expected *a priori*. [6] Practically Faraday waves represents a new and versatile tool to explore some complex problematics of turbulence such as particle dispersion [5] or the emergence of large scale coherent structures via spectral condensation [7].

References

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