THE BASIC PHYSICS OF THE LINEAR TRANSIENT GROWTH IN PLANE SHEAR FLOWS

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Abstract The energy transient growth mechanism of linear perturbations in plane constant shear flows is re-examined. Considering fluid particle dynamics and operating in terms of the pressure force, we focus on the physics of the energy exchange between the base flow and a single Kelvin mode (i.e. plane waves or spatial Fourier harmonics of perturbations). The keystone of the energy exchange physics is the elastic reflection of the fluid particles from the maximum pressure plane of the Kelvin mode. An interplay of these physics with the shear flow kinematics quantitatively exactly describes the transient growth and, what is most important, the linear dynamics of the system allows to construct the dynamical equations that are identical to the Euler ones. The proposed mechanism is equally applicable to two- and three-dimensional (2D and 3D) perturbations and, thus, shows the universal nature of the transient growth physics in contrast to the widely accepted explanations, separating 2D (Orr mechanism) and 3D perturbations (lift-up mechanism).

INTRODUCTION

In the 1990s the non-normal nature of nonuniform/shear flows was rigorously and finally revealed [1, 5, 6] that caused the breakthrough in the understanding of linear and nonlinear shear flow dynamics. Yet, a central question arises: What is the physical specificity of these flows? The fact is that shear flows do not directly bring any force forth. However, a force is induced at the interplay of the background flow and the perturbations, inducing a perturbation pressure gradient that causes the dynamical activity of the flow. Herein, one circumstance is of particular importance: The shearing of the perturbations due to the background shear flow results in a time dependence of cross-stream wave-number $k_y(t)$ of the Kelvin mode that continually changes the character of the induced pressure force. This permanently changing force excludes an exponential time dependence of the dynamics and makes the spectral time analysis, at least, non-optimal. The induced pressure force has an additional peculiarity: It does not perform work. Consequently, it is absent in the energy balances of the dynamics. This circumstance makes the pressure, in some sense, "invisible", hence encourages the analysis of shear flows with the help of dynamical equations, e.g. the vorticity equations. Of course, such a form of the equations is correct, as the pressure force does not affect the vorticity dynamics. Yet, one should be aware that the main factor of the dynamical activity of shear flows – the pressure force – is missing in the vorticity equations and while analysing, the basis of energy exchange processes slips out of sight. That is why the widely accepted mechanisms of transient perturbation growth in plane shear flows – Orr [4] and lift-up mechanisms [3, 5] – are somewhat imperfect and contradictory.

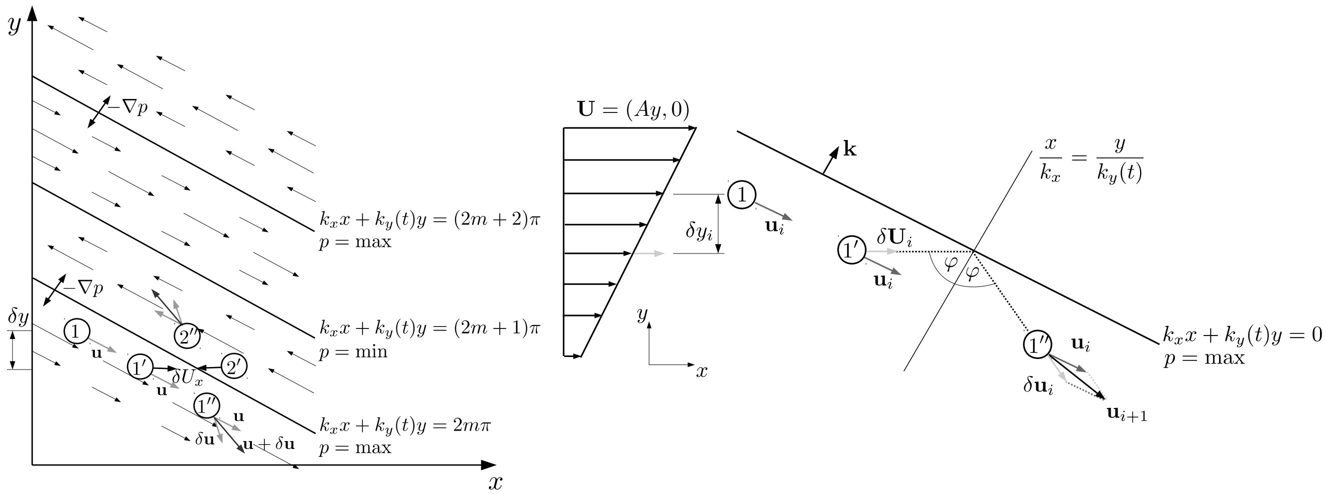
The shortcoming of these mechanisms is that they mathematically reliably describe the growth/attenuation of perturbations, yet leave the underlying physical basis/mechanism out of sight.

The contradictory of these mechanisms is that the Orr mechanism is just suitable for 2D perturbations, while the lift-up one just for 3D perturbations. In reality, the physics of the transient growth are consistent and universal and therefore equally applicable for 2D and 3D perturbations.

RESULTS

In order to solely focus on the transient growth physics in their purest form we consider an inviscid and unbounded flow with linear velocity profile. The physical mechanism of the transient growth that operates in terms of pressure forces is proposed in [2]. The considered 2D problem of the Kelvin mode dynamics in plane flows with velocity $\mathbf{U} = (Ay, 0)$ is analysed in terms of single fluid particle (SFP) dynamics. To remind: A Kelvin mode is defined by perturbation amplitudes and phases, which depend on time: $\Phi(x, y, t) = \tilde{\Phi}(t) \cdot \exp(ik_x x + ik_y(t)y)$, where Φ denotes the physical variables and $\tilde{\Phi}$ the respective amplitude, k_x the stream-wise and $k_y(t) = k_y(0) - Ak_x t$ the time dependent cross-stream wavenumber. The basis of our explanations and derivations shall be Figure 1.

The transient growth of a 2D single plane wave can be entirely comprehended inside the physical plane. As we consider a plane wave with pressure, p, and velocity fields, $\mathbf{u} = (u_x, u_y)$, having the following phases: $\Psi_p = k_x x + k_y(t)y$, $\Psi_{u_x} = k_x x + k_y(t)y + \pi/2$ and $\Psi_{u_y} = k_x x + k_y(t)y - \pi/2$, the maximum pressure equation is given by $k_x x + k_y(t)y + k_z z = 2\pi m$, where $m = 0, \pm1, \pm2, \ldots$, as depicted by the solid lines in Figure 1(a). The velocity field satisfies the incompressibility equation, i.e. $\mathbf{k}(t) \cdot \hat{\mathbf{u}} = 0$, the hat denoting the Fourier transformed, and is presented by the black arrows in the figure, directed parallel to the planes of constant phase and pressure. The forces resulting from the pressure gradient (∇p)



(a) schematic description of SFP dynamics

(b) enlarged view regarding a representative SFP

Figure 1: (a) Qualitative figure illustrating the basic mechanism of energy transfer from a shear flow to a single spatial Fourier harmonic, initially located at $k_y/k_x \gg 1$. The lines $k_x x + k_y(t)y = 2m\pi, (2m+1)\pi$ and $(2m+2)\pi$ represent the intersection of the corresponding planes of constant phase with the $z = 0$ plane. The circles $1, 1'$ and $1''$ indicate an arbitrarily chosen virtual single fluid particle (SFP) supporting the qualitative analysis at different times. (b) Simplified enlarged sketch of (a) pointing out the basic feature of the reflection at the wall of constant pressure ($p = \max$)

are directed orthogonal to the maximum pressure lines, form an impermeable pressure wall for the elastically reflecting impinging SFP with perturbation velocity $\mathbf{u}$. These constant phase lines, which are perpendicular to the wavenumber vector, rotate clockwise under the influence of the background shear (as $A > 0$). Considering Figure 1(a) the basic mechanism of transient growth/fluid particle energy growth is represented by the sequence of two elementary processes:

I. Shift/deviation of the fluid particle (due to the cross-stream perturbation velocity) from its original level y by δy to its new location (1 to $1'$) during a short period of time δt, resulting in the SFP moving $\delta \mathbf{U} = \delta U_x$ faster than the background flow in the level of the new location;

II. Due to this additional stream-wise velocity (collision-velocity), $\delta \mathbf{U}$, the fluid particle undergoes an elastic collision with the "pressure wall" ($1'$ to $1''$). The collision transforms the collision-velocity $\delta \mathbf{U}$ to the additional perturbation velocity $\delta \mathbf{u}$. For this reason, the perturbation velocity has changed to $\mathbf{u} + \delta \mathbf{u}$ after the collision.

This sequence is identical for SFP "behind" the pressure wall, namely those labelled with 2. Considering this chain of processes during a longer period of time, i.e. for many iterations, i (see Figure 1(b)), the proposed scheme constructs a quantitative exact time behaviour of all physical quantities that is identical to the linearised Euler equations. For simplicity an enlarged sketch of SFP dynamics is introduced, Figure 1(b). It is easily understandable from this sketch that $|\mathbf{u}+\delta\mathbf{u}|^2 \simeq |\mathbf{u}|^2 + |\mathbf{u}||\delta\mathbf{u}| \cos\psi$ with $\psi(t) = \measuredangle(\mathbf{u}, \delta\mathbf{u})$. Therefore, a growth in perturbation energy ($|\mathbf{u}+\delta\mathbf{u}|^2 - |\mathbf{u}|^2 > 0$) only appears when $\psi(t) < \pi/2$, while for angles of $\psi(t) > \pi/2$ the perturbation energy decreases. In the framework of this research we are able to show that: *a*) The keystone of the energy exchange physics is the elastic reflection of fluid particles from the maximum pressure plane of Kelvin mode; *b*) The interplay of this process and the shear flow kinematics quantitatively exactly describes the transient growth process including the *cause and moment* of alteration of growth and attenuation; *c*) The proposed physics allow us to construct the dynamical equations, being identical to the linear Eulerian ones. This identity uniquely proofs the basic nature and exactness of the presented mechanism, equally applicable to 2D or 3D, yet principally differing from the analysis of Orr [4] and the lift-up mechanism [5].

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