SOME STUDIES ON SPATIAL DYNAMICS OF INCOMPRESSIBLE TWO DIMENSIONAL WAKES OF CYLINDERS WITH DEFORMABLE BOUNDARIES

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Abstract A novel kind of vorticity & stream-function algorithm for planar incompressible flows with deformable boundaries is put forward based on the finite deformation theory with respect to the curvilinear coordinates corresponding to the current physical configurations including time explicitly. Its essential properties are the constructed curvilinear coordinates/diffeomorphisms through which general geometrically irregular and time varying flow domains can be transformed onto geometrically regular and time fixed parametric domains and all of the tensors with their differential operators are represented by local bases introduced naturally by the curvilinear coordinates. As some applications, the wakes of radical-oscillation circular cylinder, principle-axis-oscillation elliptic cylinder and stationary-wave-oscillation circular cylinder have been simulated and the spatial dynamics with respect to different kinds of deformable boundaries have been analyzed in the point of view of vorticity and vortex dynamics.

FORMALISM OF VORTICITY & VORTEX DYNAMICS

Recently, we developed a kind of finite deformation theory with respect to the curvilinear coordinates corresponding to the current physical configurations including time explicitly through which the geometrically irregular and time varying physical configurations can be mapped in the diffeomorphism manner to the regular and fixed domains in the parametric space [1]. The key merit of the new theory is that the generally curved and time varying boundaries in physical space can be mapped onto planes or straight lines. For two dimensional flows, this kind of curvilinear coordinates can be constructed as sketched in Fig.1.

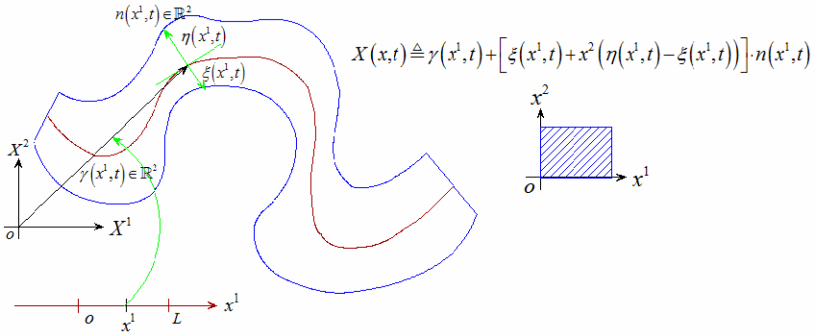

Figure 1. The sketch of general curvilinear coordinates including time explicitly for two dimensional flows on the plane with deformable boundaries, where X and x denote Cartesian and curvilinear coordinates respectively.

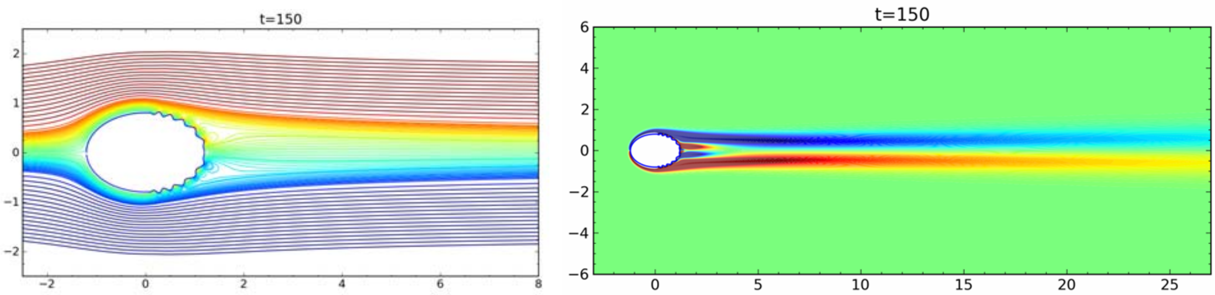

Figure 2. Snaps of flow patterns of an elliptic cylinder with traveling wave boundary. The left and right subplots correspond to the spatial distributions of stream-function and vorticity respectively in the case of Re = 500.

The corresponding parametric domain is a fixed rectangle therefore it is suitable for the numerical study on the flows on the plane around cylinders with deformable boundaries. The dimensionless component form of the governing equations of vorticity & stream-function with respect to the curvilinear coordinates including time explicitly has been deduced as following

$$\begin{cases}\dfrac{\partial\omega}{\partial t}+V^i\dfrac{\partial\omega}{\partial x^i}(x,t)-\left(\dfrac{\partial X}{\partial t}\right)^i(x,t)\cdot\dfrac{\partial\omega}{\partial x^i}(x,t)=\dfrac{1}{Re}\left(g^{ij}\dfrac{\partial^2\omega}{\partial x^i\partial x^j}-g^{ij}\Gamma_{ij}^k\dfrac{\partial\omega}{\partial x^k}\right)(x,t)\\ g^{ij}\left[\dfrac{\partial^2\psi}{\partial x^i\partial x^j}-\Gamma_{ij}^k\dfrac{\partial\psi}{\partial x^k}\right](x,t)=-\omega\end{cases},\quad \frac{\partial X^i}{\partial t}(x,t)=\left(\frac{\partial X}{\partial t}(x,t),g^i\right)_{\mathbb{R}^2}$$

Where ω is the spanwise component of the vorticity, ψ is the stream-function, g^{ij} and Γ_{ij}^{k} denote the metric tensor and Christoffel symbol respectively. On the other hand, the distribution of pressure can be determined either through the following equation

$$\Delta p = -\left[\left(V - \frac{\partial X}{\partial t}(x,t)\right)\otimes\nabla\right]:(\nabla\otimes V)$$

or the integration of Navier-Stokes equations. The validity and equivalence of both methods have been verified in the present study. As a verification of the whole algorithm, we reproduced the suppression of vortex stress of a circular cylinder by introduce certain traveling wave on the cylinder's boundary as reported firstly by C.J Wu et al [2]. Furthermore, we discovered that the same phenomenon can also occur on an elliptic cylinder as shown in Fig.2.

CONTRASTIVE STUDIES ON SPATIAL DYNAICS

The spatial dynamics of flow fields we have studied can be divided into two groups termed as *Global Dynamics* and *Local Dynamics*. The former group includes drag and left coefficients, spatial distributions of stream function, vorticity, pressure and the maximum eigen-value of strain tensor. The latter one includes boundary distributions of vorticity, pressure with their normal derivatives/fluxes and the direction of the eigen-vector of strain tensor. The flows around the radical-oscillation circular cylinder, stationary-wave-oscillation circular cylinder and principle-axis-oscillation elliptic cylinder have been simulated by the above mentioned vorticity & stream-function algorithm. And the spatial dynamics with respect to different kinds of deformable boundaries have been contrastively studied. Some properties of spatial dynamics are concluded in Table.1. Two typical flow patterns correspond to the principle-axis-oscillation elliptic cylinder and stationary-wave-oscillation circular cylinder are shown in Fig.3. That are quite differ from general wakes.

Fable 1. Summary of some global & local dynamics of flows around cylinders with different deformable boundaries.

	Global Dynamics					Local/Boundary Dynamics			
Cylinder Type	Flow Patterns	C_D			C_f	Shear Stress	Vorticity	Vorticity Flux	Major Angle of Strain
		Mean value	Amplitude	Spectrum Analysis	Spectrum Analysis	Time domain analysis	Spectrum Analysis	Time domain analysis	Time domain analysis
SCC	shedding	1.35	0.14	f_0	F_0	local varying	f_0	local varying	45°
ROCC	shedding	2.05	9.50	pf_0	f_0 ; $pf_1 \pm f_0$	global qusi-periodic varying	f_0 ; pf_1; $pf_1 \pm f_0$	Semi-cylinder qusi-periodic varying	45°
POEC	tearing, shedding	2.31	6.52	pf_0	f_0 ; $pf_1 \pm f_0$	global qusi-periodic varying	f_0 ; pf_1; $pf_1 \pm f_0$	global qusi-periodic varying	varying with deformation
WOCC	tearing, merging, shedding	1.73	3.06	pf_0	f_0 ; $pf_1 \pm f_0$	global qusi-periodic varying	f_0 ; pf_1; $pf_1 \pm f_0$	global qusi-periodic varying	varying with deformation

Remark: SCC referred to stationary circular cylinder; ROCC to radical-oscillation circular cylinder; POEC to principle-axis-oscillation elliptic cylinder; WOCC to stationary-wave-oscillation circular cylinder. f_0 & f_1 denote shedding frequency and boundary oscillation frequency respectively, p denotes nature numbers.

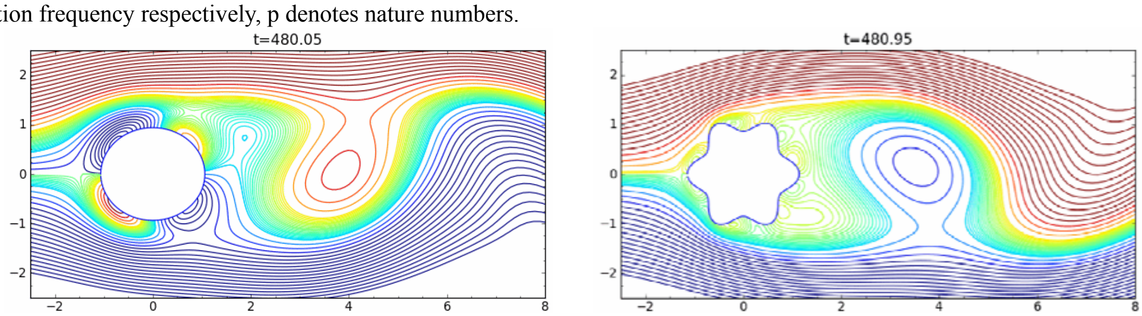

Figure 3. Snaps of flow patterns/distributions of stream-function corresponding to the principle-axis-oscillation elliptic cylinder (left) and stationary-wave-oscillation circular cylinder (right) respectively in the case of Re = 500.

References

[1] X.L. Xie, Y. Chen, and Q. Shi. Some studies on mechanics of continuous mediums viewed as differential manifolds. *Science China G* **56**: 1–25, 2013.

[2] C.J. Wu, L. Wang and J.Z. Wu. Suppression of Von Karman vortex street behind a circular cylinder by traveling wave generated by a flexible surface. *Journal of Fluid Mechanics* **574**: 365–391, 2007.