ONSET OF CHAOTIC MIXING IN A TWO-DIMENSIONAL DIFFERENTIALLY HEATED CAVITY

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Abstract We investigate numerically the Lagrangian mixing of passive tracers inside a two-dimensional differentially heated cavity filled with air. The onset of chaotic mixing corresponds to the loss of integrability of the Hamiltonian system governing the dynamics of the particles. It is found to correspond to the value of the Rayleigh number $Ra = Ra_c$ where the Eulerian flow undergoes a Hopf bifurcation. A Melnikov function is computed to identify the emergence of a homoclinic tangles. Other mixing regions emerge due to the breakdown of invariant tori due to K.A.M. resonances as Ra is increased. The evolution from incomplete towards complete mixing is investigated as a function of $Ra > Ra_c$.

Tracking of passive tracers is the key for understanding Lagrangian mixing in the limit of zero molecular diffusion. We consider here the flow of air inside a two-dimensional differentially heated cavity. The cavity has a geometrical height-to-width aspect ratio of 2, the top and bottom walls are adiabatic while homogeneous temperatures, respectively cold and hot, are imposed on both vertical walls. The flow is simulated under the Boussinesq approximation using an unsteady spectral simulation code based on Chebyshev polynomials. Numerical tracking of neutrally buoyant particles is implemented using a 4^{th} order Runge-Kutta integrator. The base flow is characterised by a steady circulation around the cavity. Once the Rayleigh number Ra is large enough, a thermally stratified core emerges, furthermore two detachment areas appear near the corners (see Fig. 1). At $Ra = Ra_c \approx 1.6 \times 10^8$, the Eulerian flow undergoes a Hopf bifurcation and bifurcates towards a time-periodic state characterised by oscillations of the two detached areas [1].

Since the geometry is two-dimensional and the flow is divergence-free, the dynamics of the particles is Hamiltonian, where the Hamiltonian H corresponds to the streamfunction [2]. Lagrangian chaos occurs precisely when the Hamiltonian system loses its integrability, i.e. for $Ra = Ra_c$. The topology of the steady base flow is first analysed with an emphasis on detecting homoclinic and heteroclinic points. For each of them, considering the time-periodic Eulerian perturbation occuring for $Ra > Ra_c$, we compute numerically the Melnikov function D. Its zeros predict multiple crossings of the associated stable and unstable manifolds responsible for the formation of a homoclinic tangle, and hence for the onset of chaotic mixing. Mixing is however spatially incomplete due to the survivance of Kolmogorov-Arnold-Moser tori within any stroboscopic Poincaré section, that act as barriers against mixing. We show how such tori are destroyed by resonances as Ra is increased. Mixing maps are constructed by seeding the flow with blobs of particles initially located near each unstable Lagrangian periodic orbit identified in the system. They demonstrate how the system evolves towards complete mixing when Ra increases, see Fig. 2. Interestingly, we conclude that predicting the onset of Lagrangian mixing and the mixing map requires the sole knowledge of the linear stability characteristics of the steady base flow.
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References

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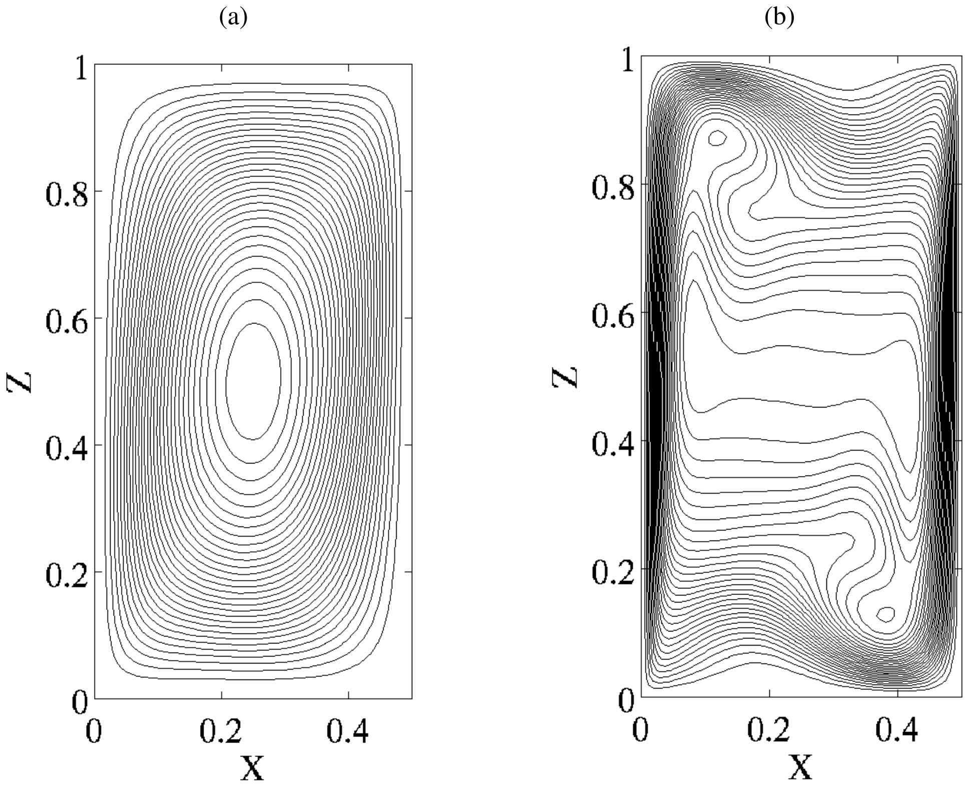

Figure 1. Streamlines of the steady states of the system for (a) $Ra = 1.0 \times 10^5$ (b) $Ra = 1.0 \times 10^7$.

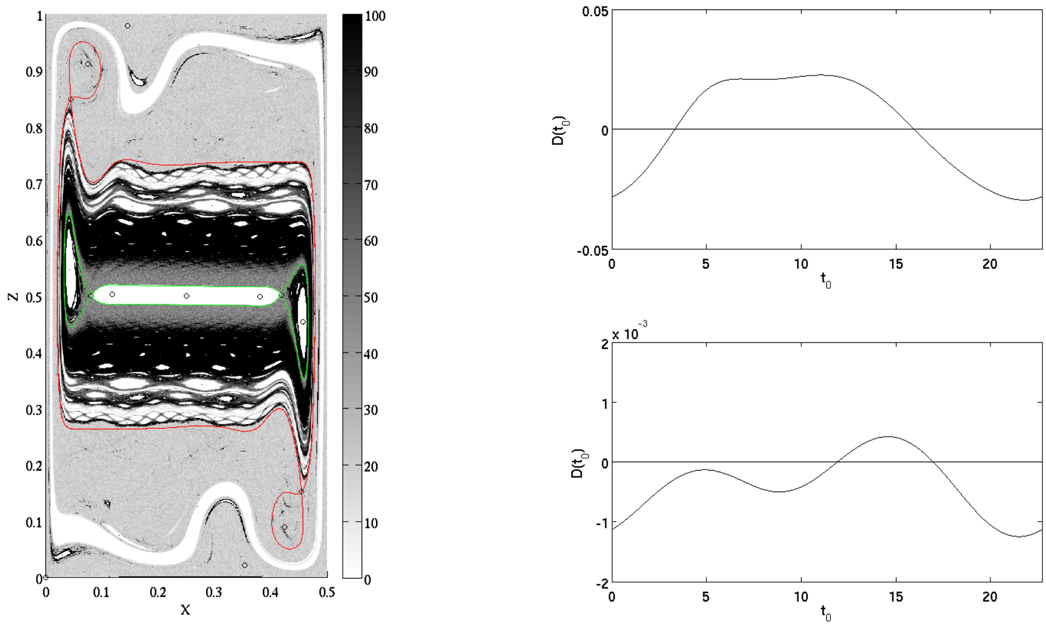

Figure 2. Left) Mixing map of the unsteady flow at $Ra = 1.625 \times 10^8 > Ra_c$. Colourbar: density of particles per small interrogation box (of size 1.0×10^{-3} per 5.0×10^{-4}) in a stroboscopic Poincaré section. Black circles: fixed points associated with the steady base state, red and green lines: homoclinic and heteroclinic loops. Right) Melnikov functions associated with the homoclinic (top) and homoclinic (center-left) loops shown in Fig. 2 (left).