LAGRANGIAN DISSIPATION DYNAMICS: SCALING STATISTICS AND PREDICTABILITY

Francois G Schmitt[1], Yongxiang Huang [2]
[1] *CNRS, Laboratory of Oceanology and Geosciences, Wimereux, France*
[2] *Shanghai Institute of Applied Mathematics and Mechanics, Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, People's Republic of China*

Abstract We consider here the Lagrangian dissipation field, using DNS data with $Re_\lambda = 400$. We first consider its statistical properties: it has a close to log-normal pdf, and it allows to directly test the Refined Similarity Hypothesis by considering scaling exponents of the Lagrangian structure functions $\zeta_L(q)$ of the velocity field, and on the other hand the Lagrangian dissipation scaling exponents $\xi_L(q)$. We find that the Kolmogorov relation in Lagrangian frame, $\zeta_L(q) = q/2 - \xi_L(q/2)$ is well verified. We then consider the log-dissipation, and find that its covariance has a logarithmic decay, as expected for a multifractal cascade process. We propose to model such series using a FARIMA truncated process, and discuss an auto-regressive representation, allowing to consider the predictability properties of dissipation Lagrangian dynamics.

LAGRANGIAN MULTIFRACTAL FRAMEWORK FOR THE VELOCITY AND DISSIPATION

One of the characteristic features of fully developed turbulence is the intermittent nature of velocity fluctuations and of the local dissipation. For Eulerian turbulence, intermittency provides corrections to Kolmogorov's scaling law, which are now well established. In the Lagrangian framework, an analogous approach can be followed. Let us note $V(t)$ the Lagrangian velocity along an element of fluid. As an analogy with Kolmogorov's dimensional analysis in the Eulerian framework, Landau proposed in his book in 1944 a $1/2$ law for the temporal increments of the Lagrangian velocity $\Delta V_\tau = |V(t+\tau) - V(t)|$. This was later generalized by Novikov, with a Lagrangian intermittency (multifractal) framework for the velocity [1, 2, 3, 4, 5]: $< \Delta V_\tau^q > \sim \tau^{\zeta_L(q)}$. For a constant dissipation one obtains the expression neglecting intermittency: $\zeta_L(q) = q/2$. In this framework, in case of intermittency $\zeta_L(q)$ is nonlinear and concave, and the non-intermittent function is valid only for $q = 2$: $\zeta_L(2) = 1$. We note the Lagrangian dissipation ε_τ (dissipation averaged during a time of τ) which is assumed to result from a multiplicative cascade and possess the multifractal scaling: $< \varepsilon_\tau^q > \sim \tau^{-\xi_L(q)}$. The phenomenological relation of Kolmogorov and Landau gives in the line of the Eulerian Refined Similarity Hypothesis (RSH) $\Delta V_\tau \sim \varepsilon_\tau^{1/2}\tau^{1/2}$, which can also be written for the scaling exponent functions:

$$\zeta_L(q) = \frac{q}{2} - \xi_L\left(\frac{q}{2}\right) \tag{1}$$

We have here $\xi_L(1) = 0$ and if the intermittency exponent is noted $\mu = \xi_L(2)$ we have $\mu = 2 - \zeta_L(4)$.

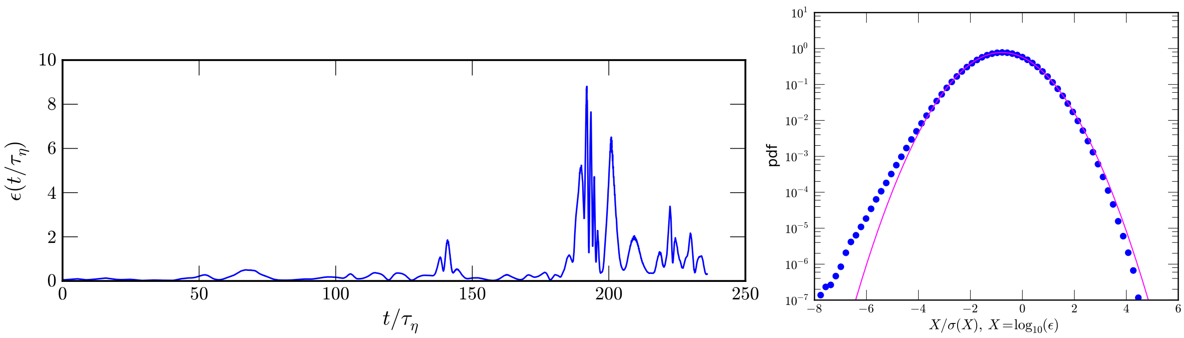

Figure 1. Left: A portion of the Lagrangian dissipation. Right: the probability density function of $\log\epsilon$, showing that the lognormal fit is rather good, except for large negative values.

DIRECT TEST OF THE REFINED SIMILARITY HYPOTHESIS USING SCALING EXPONENTS

Here we use Lagrangian velocity trajectories, with dissipation field computed along these trajectories, in an homogeneous and isotropic turbulent flow DNS simulation with $R_\lambda = 400$: see [6] for the presentation of the database. Ensemble statistics are computed over $200,000$ trajectories. A portion of the dissipation field along a trajectory is shown in Fig.1a. The probability density function of $\log\epsilon$ is shown in Fig.1b, and compared to a Gaussian fit. This shows that the log-normal classical approximation is here rather justified, except for large negative values. Scaling exponents are estimated

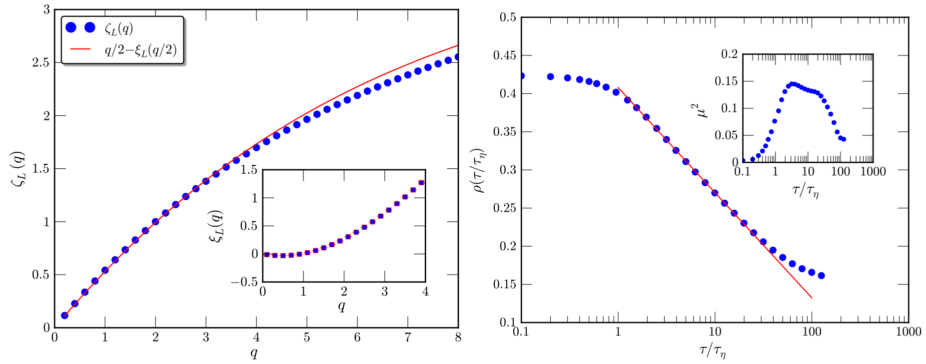

Figure 2. Left: comparison of the Lagrangian velocity structure functions scaling exponents $\zeta_L(q)$ with $q/2-\xi_L(q/2)$: the good superposition confirms the Refined Similarity Hypothesis in the Lagrangian frame, using directly scaling exponents. Right: the covariance of $g = \log\epsilon$, in log-lin plot. For multifractal cascades, this is expected to be proportional to $\log\tau$, which is verified here.

separately for the velocity (to compute $\zeta_L(q)$) and for the dissipation field using coarse graining (to compute $\xi_L(q)$). The result is shown in Fig. 2a: the good superposition confirms the RSH in the Lagrangian frame, using directly scaling exponents. The RSH was already verified in [6] and [7], but using different approaches, for selected moments orders, or using correlations. Fig. 3a is the first direct check of this hypothesis for scaling exponents.

MODELLING OF THE LOG-DISSIPATION AND PREDICTABILITY STUDY

The covariance of $g = \log\epsilon$, where ϵ is obtained from a multifractal cascade, is proportional to $\log\tau$ (see [8] and references therein). This property is often taken as a validation of the multifractal cascade framework. Here this covariance was computed from the DNS data, and shown in Fig.2b in log-lin plot. The good straight line which is obtained shows that the Lagrangian dissipation can be considered as the result of a multifractal cascade. More precisely, the covariance is expected to behave as $-\mu^2\log\tau$, and the coefficient found here gives $\mu \simeq 0.37$, which is slightly larger than the value obtained from the value of $\zeta_L(4)$, which is $\mu \simeq 0.3$.

In [8], a discrete lognormal process is introduced to sequentially generate a lognormal multifractal time series. It is written as a truncated log-FARIMA discrete process, or a truncated FARIMA process for g. It was shown that such process can be also written in an autoregressive way, of the form:

$$h_t = \delta + \sum_{k\leq 0} d_k g(t-k) \tag{2}$$

where h_t is a noise term, δ is a constant and d_k are parameters which are known recursively. Such auto-regressive expression is useful for prediction purpose since it shows that, when the past values of g are known, the value of $g(t)$ is provided with the help of only one noise term h_t. Introducing such modelling for the logarithm of the Lagrangian dissipation field, one can express $\log\epsilon(t)$ using past values of ϵ and only one noise term. This autoregressive expression fully exploits the long-range correlations of the intermittent dissipation field. We will show, using the DNS database, the predictability strength of such modelling.

References

[1] E. A. Novikov. Two-particle description of turbulence, Markov property and intermittency. *Phys. Fluids A* **1**, 326 (1989).
[2] M. S. Borgas. The multifractal Lagrangian nature of turbulence. *Philos. Trans. R. Soc. London, Ser. A*, **342**, 379-411, 1993.
[3] H. Xu, M. Bourgoin, N. T. Ouellette, and E. Bodenschatz. High order Lagrangian velocity statistics in turbulence. *Phys. Rev. Letters* **96**, 024503, 2006.
[4] Schmitt, F. G. Linking Eulerian and Lagrangian structure functions' scaling exponents in turbulence. *Physica A* **368**, 377-386, 2006.
[5] L. Biferale, E. Bodenschatz, M. Cencini, A.S. Lanotte, N.T. Ouellette, F. Toschi, and H. Xu. Lagrangian Structure Functions in Turbulence: A Quantitative Comparison between Experiment and Direct Numerical Simulation. *Phys. Fluids* **20**, 065103, 2008.
[6] R. Benzi, L. Biferale, E. Calzavarini, D. Lohse and F. Toschi. Velocity-gradient statistics along particle trajectories in turbulent flows: The refined similarity hypothesis in the Lagrangian frame. *Phys. Rev. E* **80**, 066318, 2009.
[7] H. Yu and C. Meneveau. Lagrangian Refined Kolmogorov Similarity Hypothesis for Gradient time Evolution and Correlation in Turbulent Flows. *Phys. Rev. Letters* **104**, 084502, 2010.
[8] N. Perpete and F.G. Schmitt. A discrete log-normal process to sequentially generate a multifractal time series. *Journal of Statistical Mechanics: Theory and Experiement* **P12013**, 2011.