REGULARIZED GAS DYNAMIC EQUATIONS FOR NUMERICAL SIMULATION OF COMPRESSIBLE TURBULENT FLOWS

T. G. Elizarova[1] & I. A. Shirokov [2]
[1]*Keldysh Institute for Applied Mathematics Russian Academy of Sciences, Moscow, Russia*
[2]*Moscow State University, Moscow, Russia*

Abstract The possibilities of the regularized (or quasi-) gas dynamic equation system in numerical simulation of compressible turbulent subsonic flows is shown for the problem of the uniform isotropic turbulence decay.

In this paper we show the possibilities of the regularized (or quasi-) gas dynamic (QGD) equation system in numerical simulation of compressible turbulent subsonic flows. The background, finite-difference approximations and various applications of the QGD system are presented in monographs [1]-[3]. A number of theoretical results including Petrovskii parabolicity and linearized stability of equilibrium solutions of QGD system have been established in, e.g. [4].

QGD equations can be obtained by averaging gas dynamics equations over a small time interval, which results in additional smoothing or regularization. The additional strongly non-linear terms that appear due to averaging are second-order space derivatives in factor of a small parameter τ that has the dimension of a time. The influence of additional terms is inessential for the stationary flows, but for strongly nonstationary flows their contribution becomes important. The τ-terms bring an additional entropy production and so they have a dissipative character. In the common notations the τ-addition to the dissipative function writes

$$\Psi_\tau = \frac{\tau p}{\rho^2}\Big(div(\rho\vec{u})\Big)^2 + \frac{\tau}{\rho}\Big(\rho(\vec{u}\cdot\nabla)\vec{u} + \nabla p\Big)^2 + \frac{\tau}{\rho\epsilon}\Big(\rho(\vec{u}\cdot\nabla)\epsilon + p div\vec{u}\Big)^2.$$

In calculations parameter τ is defined as $\tau = \alpha h/c$, where h is the mesh size, $c = \sqrt{\gamma p/\rho}$ is the speed of sound, $0 < \alpha < 1$ is a numerical coefficient to be tuned, and h/c is the time required for a perturbation to travel across a grid cell. So τ-terms reproduce a non-trivial kind of subgrid-type dissipation. Similar to sub-grid dissipation in LES models, it smoothes or averages the fluctuations of flow parameters on a time-space scale depending on discretization. The subgrid dissipation in QGD equations differs from the turbulent Smagorinsky viscosity [5], as the τ-terms have different mathematical structure and properties. Additional terms appear not only in the momentum and energy equations, but also in the continuity equation. This latter property models the turbulent mass-diffusion, which is inherent to turbulent mixing. Along a wall, the τ-terms vanish.

These features of the QGD equations open nice perspectives for the simulation of turbulent flows. The first encouraging results for laminar-turbulent transition in a separated flow over a backward-facing step [6] and in a vicinity of a hypersonic vehicle [7] were obtained.

Here we show the first results of numerical simulation of a uniform isotropic turbulence decay. As an initial condition we take Orzag-Tang vortex in the L-dimension cube, where gas pressure and density are constant, and velocity components are

$$u_x = U sin(2\pi y/L), \quad u_y = -U sin(2\pi z/L), \quad u_z = U sin(2\pi x/L),$$

Mach number $Ma = 0.2$, Reynolds number $Re = 30000$. Problem is solved with periodic boundary conditions. Here $\alpha = 0.1$ and space grid 64x64x64. Calculations are performed with gas dynamic equations in dimensional form for Nitrogen.

The initial distribution of the kinetic energy $E = \rho\vec{u}^2/2$ is shown in the first figure. Other figures consequently show the temporal evolution of the kinetic energy, that decays according with the dissipation inherit to the QGD system. The evolution of the energy spectrum $E(k)$ with time is demonstrated in the last figure. Here the Kolmogorov - Obukhov law in the spectral form $E(k) \sim k^{-5/3}$ is shown for the comparison. The evolution of the kinetic energy distribution with time from one-mode picture to a complicated pattern with a spread number of frequencies and formation of the Kolmogorov -like spectrum with increasing of time is obtained.

In the full paper the QGD system and numerical algorithm will be discussed together with the investigations of the flow with adjustments of the tuning coefficient α and the influence of the initial conditions and Reynolds number.

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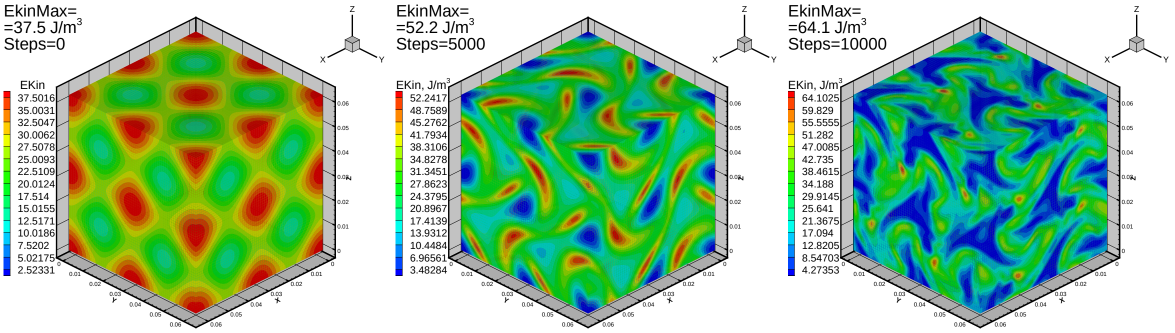

Figure 1. Time-evolution of kinetic energy

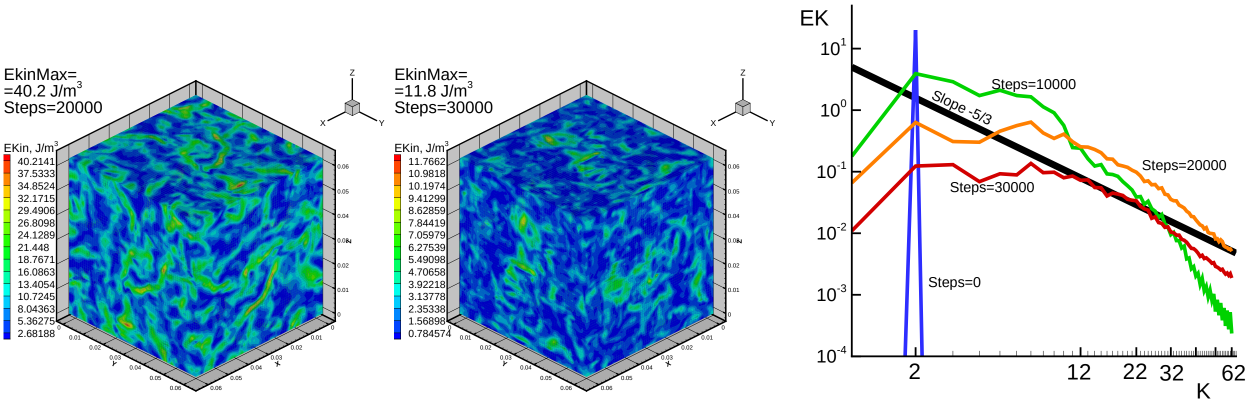

Figure 2. Time-evolution of kinetic energy and spectrum