WAVEGUIDE MODEL OF COHERENT STRUCTURES IN THE DEVELOPED TURBULENT BOUNDARY LAYER

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Abstract The nonlinear equation for Fourier components of vertical velocity of Tollmien-Schlichting (TS) waves is reduced from Navier-Stokes equations. It describes incompressible fluid pulsations in boundary layer on a plate at zero pressure gradient up to the third order of amplitude in the one-mode approximation. The wave amplitudes are represented as the sum of coherent and stochastic parts which are governed by system of equations with a small parameter ε. The multiscale method is used for solving of this system. Multiple 3-wave resonance equation is obtained for coherent structure amplitudes in the scale τ_1 ($\tau_1 = \tau_0/\varepsilon$, $\tau_0 = \delta^{**}/U_\infty$) . The integrodifferential close equation for the two-point correlation function of the incoherent part is given in the scale τ_2 ($\tau_2 = \tau_0/\varepsilon^2$). This equation contains the source member specified by the coherent structure. The closure of the moment equation chain is defined by the existence of the small parameter ε .

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The early high-accuracy experimental studies [1] have shown the developed turbulent boundary layer contains the organized vortical structures which define many physical properties of these flows. Last experimental [2] and numerical [3] researches confirm the existence of the coherent structures. It is interesting to construct a simplified mathematical model for this phenomenon from the first principles. The waveguide model of developed boundary layer [4] is one of the meaningful approach to solve this problem. By analogy to this model the nonlinear equation for Fourier components of the vertical velocity of Tollmien-Schlichting (TS) waves was obtained [5] from Navier-Stokes equations for pulsations in the boundary layer up to third power by amplitude at one-mode approximation. The equation contains the small parameter $\varepsilon^2 \sim \delta^{**}/U_\infty$ where δ^{**} is the momentum thickness, U_∞ is the free stream velocity. The interrelation (scaling law) $\delta^{**}/L \sim |\mathrm{Im}[\omega^*(\mathbf{k}, \delta^{**})]|$ introduced for matching of the equation members, where $|\mathrm{Im}[\omega^*(\mathbf{k}, \delta^{**})]|$ is the minimum, by **k,** decrement of the main mode of TS – wave. The amplitudes of the waves represented as the sum of coherent and incoherent parts governed by system of equations with small parameter ε.

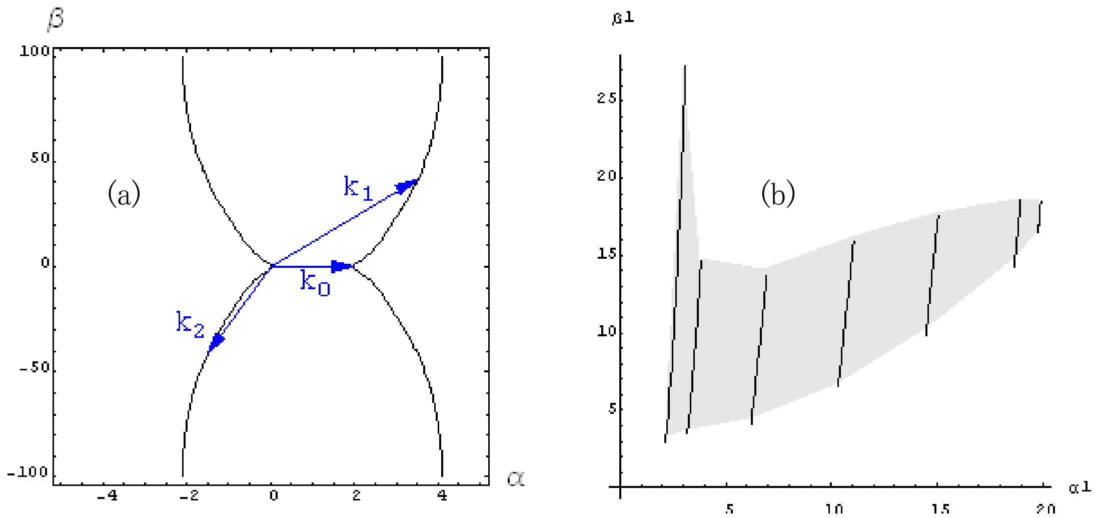

Figure 1. (a) The 3-wave resonance curve, $\mathbf{k}_i = (\alpha_i, \beta_i)$, $i = 0,1,2$. (b) The region of the wave vectors $\mathbf{k}_1$ where the weighting factors are positive (at different α_0).

The multiscale method [6] was used for solving this system ($\tau_0 = \delta^{**}/U_\infty$, $\tau_1 = \tau_0/\varepsilon$, $\tau_2 = \tau_0/\varepsilon^2$). As a result the equation of multiple 3-wave resonance was obtained for the coherent structure at τ_1–scale. A 3-wave resonance curve ($\omega_r(\mathbf{k}_0) = \omega_r(\mathbf{k}_1) + \omega_r(\mathbf{k}_2)$, $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$, $\omega_r = \mathrm{Re}(\omega)$, ω is the eigenvalue of the main mode of TS – waves) is depicted upon the Figure 1(a) and the picture shows the appearance of longitudinal vortices in the turbulent boundary layer flow. For the two-point correlation function of the incoherent part we got closed integrodifferential equation in the τ_2 – scale. The closure of the chain of the multipoint correlation equations is defined by the existence of small parameter ε [6]. This equation contains the source member defined by the coherent structure.

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It is shown that dynamics of multiple 3-wave resonance system in the discrete representation of *n* triplets satisfies invariant quadratic form of complex wave amplitudes with real weighting factors. If the weighting factors are positive the system is governed by finite motion at the surface of the unit sphere of $2(1+4n)$ – dimension at proper normalization of the amplitudes. The normalization of the quadratic form is possible due to the existence of the transformation of amplitudes and time which leave the multiple 3 – wave resonance equation unvaried. The factor of this transformation satisfies the equation at τ_2 – scale. The numerical analysis on the basis of the Musker profile of the longitudinal velocity of the turbulent boundary layer shows the existence of the wave number region were these weighting factor are positive. The wave number region of inclined waves where these factors are positive is shown in Figure 1(b). Therefore this region can be brought in correspondence with the coherent structure. Comparison of the scaling law with experimental data [7] at the range of Reynolds number $R_\delta = 10^4 - 10^5$, δ is the boundary layer thickness by velocity, shows good coincidence Figure 2. Mean values, $\langle uv \rangle_{t1}$, $\langle uu \rangle_{t1}$, $\langle vv \rangle_{t1}$, $\langle ww \rangle_{t1}$, here $t_1 = t/\tau_1$, and *u*, *v*, *w*, are components of velocity defined by the coherent structure, are defined too.

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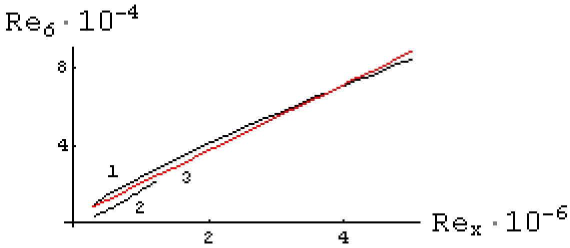

Figure 2. Comparison of the scale law with experimental data: 1, 2 – [7], 3 – the present theory.

References

[1] Belotserkovskii O.M., Khlopkov Yu..I., Zharov V.A., Gorelov S.L., Khlopkov A.Yu. Organized structures in turbulent flows. Analysis of experimental works devoted to boundary layer. M.: MIPT, 2009. – 302p.
[2] Borodulin V.I., Kachanov Y.S., Roschektayev A.P. Experimental detection of deterministic turbulence // Journal of Turbulence. – Vol. 12, N. 23. – 2011. – P. 1–34.
[3] G Khujadze, R Nguyen Van Yen, K Schneider, M Oberlack and M Farge Coherent vorticity extraction in turbulent boundary layers using orthogonal wavelets. In 13th European Turbulence Conference. Warsaw, 12-15 September 2011. University of Warsaw, Poland. Book of abstracts.
[4] Landahl M.T. A wave-guide model for turbulent shear flow. J.Fluid Mech., 1967, vol.. 29, pt. 3, p. 441. – 459.
[5] Zharov V.A. Weakly nonlinear pulsation models in laminar and turbulent boundary layers. West-East High Speed Flow Field Conference 19-22 November 2007 Moscow, Russia.
[6] Davidson R.C. *Method in nonlinear plasma theory,* N.Y.; L.:Acad. Press, (1972). (Pure and applied physics; Vol.37).
[7] Schlichting H. Boundary layer theory. M.: Nauka, 1974.