AN ANALYSIS ON BUDGET TERMS OF THE VOLUME AVARAGED REYNOLDS STRESS TRANSPORT EQUATION FOR FLOWS IN POROUS MEDIA

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Abstract To understand the turbulent flows in porous media, LES has been performed by a multiple-relaxation-time lattice Boltzmann method. Using the simulation data, the mechanism of turbulent flows is discussed by investigating the budget terms of the volume averaged Reynolds stress equation. The behaviors of production terms, pressure correlation terms and dissipation terms are analyzed by changing the porosity of the porous media. It is found that there is a relation between the porous characterisitc scales and production terms that contribute to micro-scale stresses.

OUTLINE OF NUMERICAL METHOD

Turbulent flows in porous media are often encountered in environmental and engineering fields. Although the turbulent flows play important role in heat and mass transfer, from the engineering view point, direct prediction of the turbulence in porous media is difficult due to their complex geometry. Hence, to predict the engineering turbulence in porous media, macroscopic RANS modelling is needed. In order to perform RANS modelling, it is essential to understand how each process in averaged equations works. In this study, LES is performed to discuss the mechanism of the turbulence based on the double averaged equations.

The present LES is performed by the D3Q27 MRT-LBM. The SGS model used is the WALE model which satisfies the near wall scaling; $\nu_t = o(y^3)$ without damping functions. The total viscosity: $\nu = \nu_0 + \nu_t$, where ν_0 and ν_t are the kinetic and SGS eddy viscosities respectively, are jointly applied to the relaxation factors in the MRT-LBM. In this study, the geometry illustrated in Fig.1 is considered. The computation domain is $4H(x) \times 2H(y) \times 0.5H(z)$ and periodical boundary conditions are imposed in the$x-, y-, z-$directions. Varying the square rods size D, the porosity φ of the flow field is changed: $\varphi = 1 - (D/H)^2$. The Reynolds number is defined as $\mathrm{Re}_d = U_d D/\nu_0 = 3500$, where U_d is Darcy velocity. The grid node number is $480(x) \times 240(y) \times 60(z)$.

Following Whitaker[S.Whitaker, The Forchheimer equation: A theoretical development. Transp.Porous Med. **25**, 27-61], the volume averaging is applied to the Reynolds stress transport equation. The value ϕ is decomposed into the intrinsically averaged value $\langle\phi\rangle^f$ and its dispersion $\tilde{\phi}$. The volume averaged stress tensor $R^A_{ij} = \langle R_{ij}\rangle^f$ is divided into the macro scale stresses $R^M_{ij} = \overline{\langle u'_i\rangle^f \langle u'_j\rangle^f}$ and the micro-scale stresses $R^m_{ij} = \overline{\langle \tilde{u}'_i \tilde{u}'_j\rangle^f}$. With the assumption that the porous media is homogeneous and the flow inside the porous medium is developed, the remaining terms are the production, pressure correlation and dissipation terms. They are:

$$\begin{aligned}\langle P_{ij}\rangle^f = P^A_{ij} \;=\;& \underbrace{-R^M_{ik}\frac{\partial\langle U_j\rangle^f}{\partial x_k} - R^M_{jk}\frac{\partial\langle U_i\rangle^f}{\partial x_k}}_{P^{MS}_{ij}} \underbrace{-R^m_{ik}\frac{\partial\langle U_j\rangle^f}{\partial x_k} - R^m_{jk}\frac{\partial\langle U_i\rangle^f}{\partial x_k}}_{P^{mS}_{ij}} \underbrace{- \left\langle \overline{\langle\tilde{u}'_j\tilde{u}'_k\rangle}\frac{\partial\langle\tilde{U}_i\rangle^f}{\partial x_k}\right\rangle^f - \left\langle \overline{\langle\tilde{u}'_k\tilde{u}'_i\rangle}\frac{\partial\langle\tilde{U}_j\rangle^f}{\partial x_k}\right\rangle^f}_{P^{md1}_{ij}} \\ & \underbrace{- \left\langle \overline{\langle u'_i\rangle^f\tilde{u}'_k}\frac{\partial\langle\tilde{U}_j\rangle^f}{\partial x_k}\right\rangle^f - \left\langle \overline{\langle u'_j\rangle^f\tilde{u}'_k}\frac{\partial\langle\tilde{U}_i\rangle^f}{\partial x_k}\right\rangle^f - \left\langle \overline{\langle u'_k\rangle^f\tilde{u}'_i}\frac{\partial\langle\tilde{U}_j\rangle^f}{\partial x_k}\right\rangle^f - \left\langle \overline{\langle u'_k\rangle^f\tilde{u}'_j}\frac{\partial\langle\tilde{U}_i\rangle^f}{\partial x_k}\right\rangle^f}_{P^{md2}_{ij}}, \end{aligned} \tag{1}$$

$$\langle \Pi_{ij}\rangle^f = \Pi^A_{ij} = \underbrace{\overline{\left\langle \frac{\tilde{p}'}{\rho}\left(\frac{\partial\tilde{u}_i{}'}{\partial x_j} + \frac{\partial\tilde{u}_j{}'}{\partial x_i}\right)\right\rangle^f}}_{\Pi^M_{ij}} + \underbrace{\overline{\frac{\langle p'\rangle^f}{\rho}\left(\frac{\partial\langle u'_i\rangle^f}{\partial x_j} + \frac{\partial\langle u'_j\rangle^f}{\partial x_i}\right)}}_{\Pi^m_{ij}}, \tag{2}$$

$$\langle \varepsilon_{ij}\rangle^f = \varepsilon^A_{ij} = \underbrace{\overline{\left\langle \nu\frac{\partial\tilde{u}_i{}'}{\partial x_k}\frac{\partial\tilde{u}_j{}'}{\partial x_k}\right\rangle^f}}_{\varepsilon^M_{ij}} + \underbrace{\nu\overline{\frac{\partial\langle u'_i\rangle^f}{\partial x_k}\frac{\partial\langle u'_j\rangle^f}{\partial x_k}}}_{\varepsilon^m_{ij}}. \tag{3}$$

The first term of Eq.(1) P^{MS}_{ij} is called the macro-scale mean shear production and also appears in the transport equation of R^M_{ij}. The other terms are the micro-scale mean shear production $P^{mS}_{ij}$and the dispersive shear production terms $P^{md1}_{ij}, P^{md2}_{ij}$. They appear in the transport equation of R^m_{ij}. The first term of Eq.(2): Π^M_{ij}, is called the macro-scale pressure strain that redistributes the mcaro-scale stresses R^M_{ij} and the second term Π^m_{ij} is called the micro-scale pressure strain that appears in the micro-scale stresses R^m_{ij} transport equation. In this study the REV(representative elementary volume) is considered as shown in Fig.1.

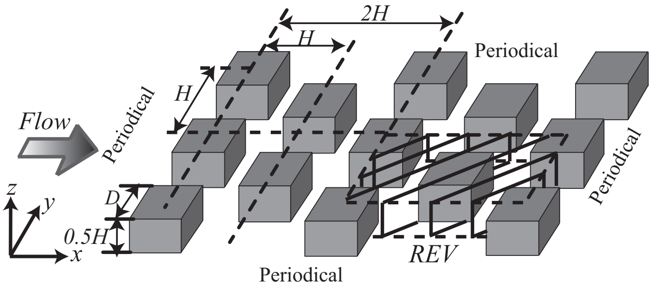

Fig.1 Computational domain.

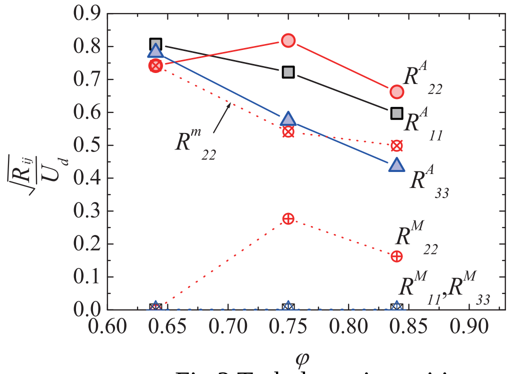

Fig.2 Turbulence intensities.

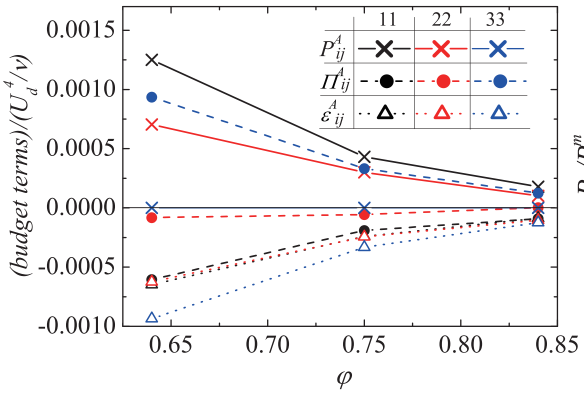

Fig.3 Budget terms.

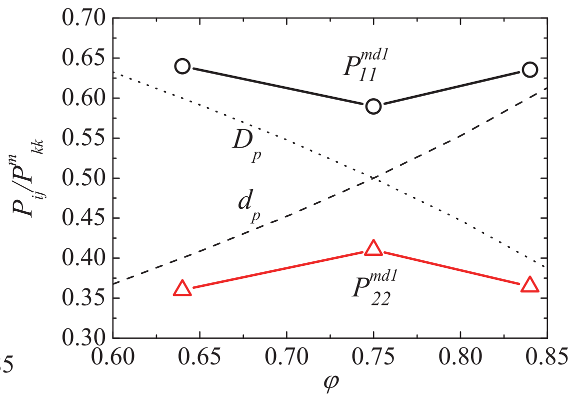

Fig.4 Production terms and length scales.

RESULTS AND DISCUSSIONS

Fig.2 shows the results of R^M_{ij}, R^m_{ij} and R^A_{ij} against the porosity φ. It can be seen that all the macro-scale stresses R^M_{ij} vanish at $\varphi = 0.64$. Although all the total turbulence intensities become smaller as the porosity becomes higher, interestingly, R^A_{22} is enhanced in the higher porosity cases. In the higer porosity cases, it is found that only the macro-scale stress R^M_{22} is produced. Consequently, $R^A_{11} = R^m_{11}$ and $R^A_{33} = R^m_{33}$ at $\varphi = 0.75$ and 0.84, whilst $R^A_{ij} = R^m_{ij}$ at $\varphi = 0.64$.

Fig.3 shows the budget terms. Since there is no gradient of the volume averaged mean velocity, the mean shear production termsP^{MS}_{ij} and P^{mS}_{ij} vanish, whilst dispersive shear production terms P^{md1}_{ij} and P^{md2}_{ij} exist. Although P^{md2}_{ij} contributes a little due to large scale perturbation, it is found to be negligibly small. Hence it is considered that P^{md1}_{ij} is the main production of R^m_{22} and $P^A_{11} \simeq P^{md1}_{11}$, $P^A_{22} \simeq P^{md1}_{22}$,$P^A_{33} \simeq 0$. Since the production of the macro-scale stress becomes 0, the macro-scale stress R^M_{22} is considered to be produced by the cascade process from the micro-scale turbulence energy.

The macro-scale pressure correlation Π^M_{ij} seems to vanish statistically, whilst it is not obvious by its definition. Thus, $\Pi^A_{ij} = \Pi^m_{ij}$. No pressure correlation of the macro-scale stress means that R^M_{11}, R^M_{33} are not redistributed from R^M_{22}. This is also confirmed by Fig.2. From these results, the micro-scale stresses R^A_{11}, R^A_{22} are produced by the production termsP^m_{ij} and R^A_{33} is mainly redistributed fromR^A_{11} by Π^m_{33}.

Fig.4 shows the production terms normalized by twice of the production of the micro-scale turbulence energyP^m_{kk}. The scales of the mean particle diameter $D_p = D$ and the mean pore diameter $d_p = 1 - D$ are also plotted. It is found that P^{md1}_{11} shows the distribution profile along with $\max(D_p, d_p)$ whilst P^{md1}_{22} accords with $\min(D_p, d_p)$. It is thus confirmed that the characteristic length scales of porous media have a relation with turbulent length scale and production terms.

CONCLUSIONS

To understand the mechanism of turbulence for RANS modelling, the budget terms of the volume averaged Reynolds stress equation are investigated by performing LES for flows in homogeneous porous media. It is found that the micro-scale stresses are produced by the production terms except for R^m_{33} and the redistribution terms are $\Pi^m_{11} \simeq -\Pi^m_{33}, \Pi^m_{22} \simeq 0$. Though the macro-scale stressR^M_{22} is produced in a higher porosity case, it is found that $R^M_{11} \simeq R^M_{33} \simeq 0$. Since the production terms of the macro-scale stresses becomes 0 and there is no pressure-correlation, the macro-scale stresses are considered to be produced by back scattering from the micro-scale stresses. It is confirmed that there is a relation between the characteristic length scales of porous media and the production terms.