STOCHASTIC EVOLUTION OF SMALL-SCALE VELOCITY FLUCTUATIONS IN HYDRODYNAMIC TURBULENCE

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Abstract Formation of vortex filaments (regions of high vorticity) is derived based on the Navier-Stokes equation. Stochasticity is introduced via random large-scale velocity components, without using external random forces. It is shown that, under general assumptions, high-vorticity regions undergo exponential stretching. The motion inside a filament is reduced to one-dimensional.

Understanding of statistical properties of a turbulent flow is one of the classical problems of hydrodynamics. In particular, experiments and numerical simulations demonstrate an intermittent behavior of velocity scaling exponents [1, 2]. The most successful, and conventional nowadays, way to interpret this intermittency is the Multifractal approach introduced in [3]. This is a generalization of Kolmogorov's K41 theory, and it allows, in particular, to express all the observed intermittent characteristics by means of velocity scaling exponents [4].

However, the Multifractal method is a phenomenological theory: it does not derive the scaling exponents from the Navier-Stokes (or Euler) equation, and it does not specify the structures, or configurations, that are responsible for the intermittency. The attempts to derive the Multifractal formalism directly from the first principles (Navier-Stokes equation) have not been successful yet.

The approach we propose in this work may help to solve the problem. In our previous papers [5, 6]we have seen that vortex filaments play the determinative role in velocity scaling. Now we investigate the stochastic Navier-Stokes (Euler) equation to retrace the formation of these filaments.

To derive statistical properties, one has to introduce randomness into the Navier-Stokes equation. This is usually done by adding a large-scale random external force (normally with Gaussian probability distribution) into the right-hand side of the Navier-Stokes equation. But the external volume-acting forces might exist in the flow and might not.

On the other hand, large-scale velocity perturbations may be themselves treated as independent random values. The stochastic properties of small-scale fluctuations can then be derived based on the properties of the large-scale fluctuations. This prevents one from adding new parameters (forces); this approach is also more general and includes the Navier-Stokes equation with external forces as a particular case.

To develop this idea, we split a velocity perturbation into large-scale and small-scale components; a large-scale component is defined as an average over some sufficiently large volume. Then we derive an equation for the evolution of small-scale perturbations.

This equation is a stochastic analog to the Navier-Stokes equation. The principal point is that large-scale velocity, not an external force, is considered as an independent random quantity.

We analyze the solutions of the equation in the vicinity of maximum of vorticity under the assumption that the large-scale velocities are Gaussian. It appears that, for a large class of solutions, the linear part of the equation acts as some permanent stretching, despite the stochastic symmetry of the large-scale velocity. This stretching produces exponential growth of vorticity and formation of vortex filaments. This stretching, and not decay, of vortices appears to be the main mechanism of turbulence development.

The nonlinear evolution of vorticity takes place against this stretching background. Then, in some special reference frame, the motion can be described by only one function of time and one space coordinate, and the equation is reduced to one differential equation of two variables. Further analysis of this equation allows to derive scaling behavior of vorticity. Taking the dispersion into account allows to obtain the intermittent scaling exponents (see the abstract by Sirota, Zybin), thus deriving Multifractal formalism from the Navier-Stokes equation.

References

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