STATISTICS OF THE VELOCITY GRADIENT TENSOR PERCEIVED BY A SET OF FOUR TRACER PARTICLES IN HOMOGENEOUS ROTATING TURBULENCE

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Abstract The dynamics and structure of rotating homogeneous turbulence is investigated through the statistical properties of the "perceived" velocity gradient tensor, defined by interpolation from the locations and velocities of a set of four particles. Results from direct numerical simulations of forced homogeneous rotating turbulence at different Rossby numbers are presented. We thus provide a multi-scale analysis of the dynamics of rotating turbulence, and of some of its important features.

DEFINITIONS

We analyze here the statistics of the velocity gradient tensor $\mathbf{M}$ perceived by a set of four points and defined as:

$$M_{ab} = \left[\boldsymbol{\rho}^{-1}\right]_{ai} v_{ib} - \frac{\delta_{ab}}{3} Tr(\boldsymbol{\rho}^{-1} \cdot \boldsymbol{v}), \tag{1}$$

where $\boldsymbol{\rho}$ (resp. $\boldsymbol{v}$) is the so-called reduced coordinates tensor (resp. reduced velocity tensor) [2]. These statistics are measured in direct numerical simulations of forced homogeneous rotating turbulence at different Rossby numbers, for isotropic tetrads ($\rho_{ab} = (r_0/\sqrt{3})\delta_{ab}$) of different sizes (between the Kolmogorov and the integral length scales, *i.e.* $\eta \lesssim r_0 \lesssim L$).

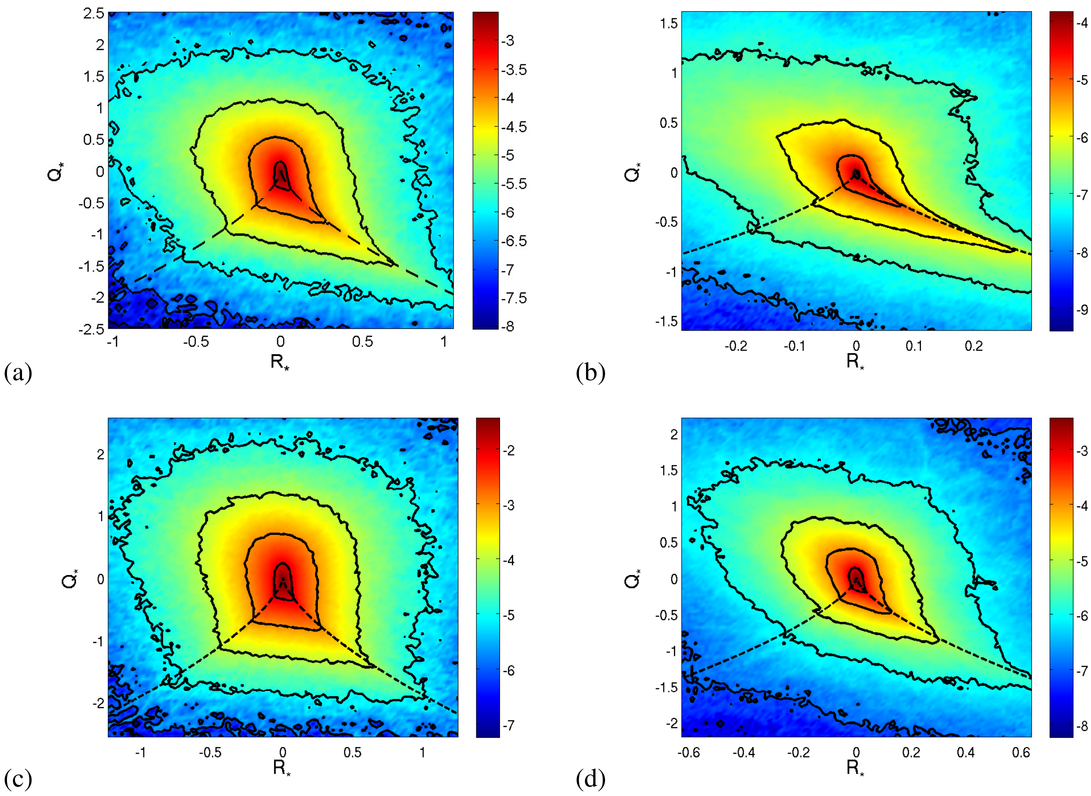

Figure 1. Joint PDF of the normalized R and Q invariants: $R_* = R/\langle (Tr(\mathbf{S}^2))^{3/2}\rangle$, $Q_* = Q/\langle Tr(\mathbf{S}^2)\rangle$. (a) Isotropic turbulence, $r_0 \approx L/3$; (b) isotropic turbulence, $r_0 \approx L/170 \approx \eta$; (c) $Ro^{(L)} = 0.07$, $r_0 \approx L/2$; (d) $Ro^{(L)} = 0.07$, $r_0 \approx L/73 \approx 1.5\eta$. The isoprobability contours represent the probabilities 10^n, where n is a negative integer (see colorbars). The dashed line is the zero-discriminant line: $27R^2 + 4Q^3 = 0$.

JOINT PROBABILITY DISTRIBUTION FUNCTIONS OF THE Q AND R INVARIANTS

A convenient representation of the local topology of an incompressible flow is the (R, Q) plane. The three eigenvalues of the 3×3 matrix $\mathbf{M}$ are indeed fully determined by the invariants $Q = -Tr(\mathbf{M}^2)/2$, $R = -Tr(\mathbf{M}^3)/3$ [1]. More specifically, if $D = 27R^2 + 4Q^3 > 0$ (the region above the zero discriminant line on Fig. 1), then two of these eigenvalues are complex conjugates, which means that the flow is locally elliptic, with locally swirling streamlines. For $D < 0$ (below this separatrix), the three eigenvalues of $\mathbf{M}$ are real: strain dominates and the flow is locally hyperbolic. Incompressibility imposes $Tr(\mathbf{M}) = 0$. For $R < 0$, two eigenvalues (or their common real part) are negative and the third one is positive, therefore the flow will be contracting in two directions ("filament-type" topology), whereas for $R > 0$ two eigenvalues (or their real part) are positive, resulting in a "sheet-like" topology of the flow.

The joint probability distribution functions (PDF) of these invariants are shown in Fig. 1, for tetrads of small and large scales, in isotropic and in rotating turbulence. In agreement with the litterature [2, 4], in the isotropic case this PDF is almost symmetric with respect to the $R = 0$ axis at large scale (Fig. 1(a)) and, for $r_0 \approx \eta$, the well-known "tear-drop shape", with an excess of probability along the $R > 0$ side of the zero-discriminant line is recovered (Fig. 1(b)). In the rotating flow (at Rossby number $Ro^{(L)} = u_{rms}/(2L\Omega) = 0.07$, where u_{rms} is the rms of the velocity fluctuations, L the integral scale, and Ω the rotation rate), the PDFs are more symmetric with respect to the $R = 0$ axis (Fig. 1(c) and (d)), i.e. closer to a Gaussian distribution [2]. In particular, we found that, in rotating turbulence, the joint PDF of R and Q at scales larger than the Zeman scale $\ell_Z = \varepsilon^{1/2}/(2\Omega)^{3/2}$ (ε is the energy dissipation rate) are much more symmetric than their counterparts in isotropic turbulence, whereas these quantities at scales smaller than ℓ_Z are qualititatively very similar to the isotropic ones [3]. For $Ro^{(L)} = 0.07$, all the scales are $> \ell_Z$ and are thus affected significantly by rotation. This is confirmed by the measurement of the skewness of R, plotted in Fig. 2 as a function of r_0 in both flows.

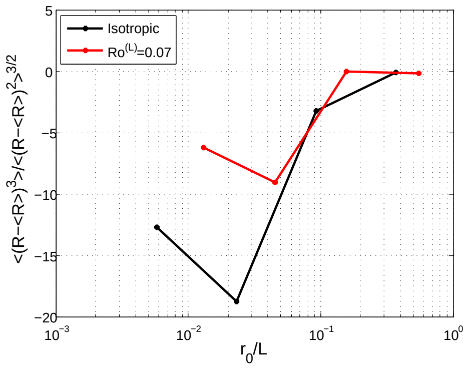

Figure 2. Skewness of R as a function of the tetrad scale r_0, in the isotropic and rotating cases.

SECOND- AND THIRD-ORDER MOMENTS OF $\mathbf{M}$

We will also present scaling laws for second- and third-order moments of the perceived velocity gradient tensor [3]. The distribution of the enstrophy and strain variance, and of their production terms, will be related to the topology of the flow, thanks to conditional probability density functions. These quantities demonstrate the role played by the Zeman scale in the elementary processes of rotating turbulence, when compared to the scale at which the perceived velocity gradient tensor is measured.

References

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